UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER Technical Report #21	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) OPERATING CHARACTERISTICS FOR AN INVENTORY MODEL THAT SPECIAL HANDLES EXTREME VALUE DEMAND		5. TYPE OF REPORT & PERIOD COVERED Technical 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Douglas Blazer		8. CONTRACT OR GRANT NUMBER(s) N00014-78-C-0467
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of North Carolina at Chapel Hill Chapel Hill, North Carolina 27514		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Mathematical and Information Sciences Division Office of Naval Research, Code 434 Arlington, Virginia 22217		12. REPORT DATE January 1983 13. NUMBER OF PAGES 15 and 74(appendices)
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Inventory Control

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this report we extend the results shown in Technical Report #19 by adding a set-up cost to the infinite horizon inventory model that special handles any demand that exceeds some value τ. We generate 2268 cases and present the operating characteristics of the special handling model as compared to the operating characteristics without special handling.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

OPERATING CHARACTERISTICS FOR AN INVENTORY MODEL THAT SPECIAL HANDLES EXTREME VALUE DEMAND

Technical Report #21

Douglas Blazer

January 1983

Work Sponsored By

Office of Naval Research (N00014-78-C0467)

Decision Control Models in Operations Research

Harvey M. Wagner
Principal Investigator
School of Business Administration
University of North Carolina at Chapel Hill

OPERATING CHARACTERISTICS FOR AN INVENTORY MODEL THAT SPECIAL HANDLES EXTREME VALUE DEMAND

Douglas Blazer

-Abstract-

In this report we extend the results shown in Technical Report #19 by adding a set-up cost to the infinite horizon inventory model that special handles any demand that exceeds some value τ. We generate 2268 cases and present the operating characteristics of the special handling model as compared to the operating characteristics without special handling.

We show that in 99.5% of the cases special handling reduces total expected costs (excluding special handling costs). Most of the cost reduction is as a result of decreased inventory investment with a smaller proportion due to decreased penalty costs and decreased set-up costs. In many cases the reduction in inventory investment significantly exceeds the amount of orders left unfilled due to special handling.

†School of Business Administration, The University of North Carolina at Chapel Hill.

TABLE OF CONTENTS

MacCormick, A. (1974), Statistical Problems in Inventory Control, ONR and ARO Technical Report 2, December 1974, School of Organization and Management, Yale University, 244 pp.

Estey, A. S. and R. L. Kaufman (1975), Multi-Item Inventory System Policies Using Statistical Estimates: Negative Binomial Demands (Variance/Mean = 9), ONR and ARO Technical Report 3, September 1975, School of Organization and Management, Yale University, 85 pp.

Ehrhardt, R. (1975), Variance Reduction Techniques for an Inventory Simulation, ONR and ARO Technical Report 4, September 1975, School of Organization and Management, Yale University, 24 pp.

Kaufman, R. (1976), Computer Programs for (s,S) Policies Under Independent or Filtered Demands, ONR and ARO Technical Report 5, School of Organization and Management, Yale University, 65 pp.

Kaufman, R. and J. Klincewicz (1976), Multi-Item Inventory System Policies Using Statistical Estimates: Sporadic Demands (Variance/Mean = 9), ONR and ARO Technical Report 6, School of Organization and Management, Yale University, 58 pp.

Ehrhardt, R. (1976), The Power Approximation: Inventory Policies Based on Limited Demand Information, ONR and ARO Technical Report 7, June 1976, School of Organization and Management, Yale University, 106 pp.

Klincewicz, J. G. (1976), Biased Variance Estimators for Statistical Inventory Policies, ONR and ARO Technical Report 8, August 1976, School of Organization and Management, Yale University, 24 pp.

Klincewicz, J. G. (1976), Inventory Control Using Statistical Estimates: The Power Approximation and Sporadic Demands (Variance/Mean = 9), ONR and ARO Technical Report 9, November 1976, School of Organization and Management, Yale University, 52 pp.

Klincewicz, J. R. (1976), The Power Approximation: Control of Multi-Item Inventory Systems with Constant Standard-Deviation-To-Mean Ratio for Demand, ONR and ARO Technical Report 10, November 1976, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 47 pp.

Kaufman, R. L. (1977), (s,S) Inventory Policies in a Nonstationary Demand Environment, ONR and ARO Technical Report 11, April 1977, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 155 pp.

Ehrhardt, R. (1977), Operating Characteristic Approximations for the Analysis of (s,S) Inventory Systems, ONR and ARO Technical Report 12, April 1977, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 109 pp.

Schultz, C. R., R. Ehrhardt, and A. MacCormick (1977), Forecasting Operating Characteristics of (s,S) Inventory Systems, ONR and ARO Technical Report 13, December 1977, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 47 pp.

Schultz, C. R. (1979), (s,S) Inventory Policies for a Wholesale Warehouse Inventory System, ONR Technical Report 14, April 1979, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 75 pp.

Schultz, C. R. (1980), Wholesale Warehouse Inventory Control with Statistical Demand Information, ONR Technical Report 15, December 1980, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 74 pp.

Ehrhardt, R. and G. Kastner (1980), An Empirical Comparison of Two Approximately Optimal (s,S) Inventory Policies, Technical Report 16, December 1980, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 22 pp.

Ehrhardt, R. (1980), (s,S) Policies for a Dynamic Inventory Model with Stochastic Lead Times, Technical Report 17, December 1980, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 20 pp.

Mosier, C. (1981), Revised (s,S) Power Approximation, Technical Report 18, February 1981, School of Business Administration, University of North Carolina at Chapel Hill, 18 pp.

Blazer, D. and M. McClelland (1981), An Inventory Model for Special Handling of Extreme Value Demands, Technical Report 19, December 1981, School of Business Administration, University of North Carolina at Chapel Hill, 10 pp.

Mitchell, J. (1982), Choosing Single-Item Service Objectives in a Multi-Item Base-Stock Inventory System, Technical Report 20, School of Business Administration, University of North Carolina at Chapel Hill, 30 pp.

We show that in 99.5% of the cases special handling reduces total expected costs (excluding special handling costs). Most of the cost reduction is as a result of decreased inventory investment with a smaller proportion due to decreased penalty costs and decreased set-up costs. In many cases the reduction in inventory investment significantly exceeds the amount of orders left unfilled due to special handling.

FOREWORD

As a part of the on-going program in "Decision Control Models in Operations Research," Mr. Douglas Blazer has extended the study on the economic impact of removing large demands in the determination of an inventory replenishment policy. He treats the multi-period case with linear costs and fixed lead-time; a set-up cost is included in this paper. Mr. Blazer derives an optimal (s,S) policy when large demand is filtered out. He compares the inventory policy cost with the filtering of large demand to the inventory policy costs for not filtering large demand. The paper provides the cost comparisons for 1440 cases. Other related reports dealing with this research program are given on the following pages.

Harvey M. Wagner
Principal Investigator

Richard Ehrhardt
Co-Principal Investigator

1. INTRODUCTION

We showed in [1] a necessary and sufficient condition for cost savings when extreme value demand is specially handled. We assumed an infinite horizon inventory model with stationary linear holding and penalty costs, yielding an optimal single critical number policy. In this report we also include a set-up cost. We test the model under various parameter settings to measure the difference between expected costs with special handling and expected costs without special handling. By definition, special handling of an extreme value demand means that existing stock is not used to satisfy the demand. The extreme value demand is either not filled at all or is filled via a special order.

1.1 The Model

We assume a single item, infinite horizon inventory model. We also assume periodic review of an item's inventory level and employ a stationary, discrete-time stochastic process to describe an item's demand. The demand sequence $q_1, q_2, \ldots,$ consists of independently, identically distributed non-negative integer valued random variables with cumulative probability $\Phi(q)$. All demands that are less than τ are met as long as stock is sufficient; when a stock-out occurs, the unfilled demand is completely back-logged. Demands greater than or equal to τ are specially handled, no existing stock is issued and these demands are not backlogged. The value of τ is assumed to be given.

Items kept in inventory are conserved, there being no losses by deterioration, obsolescence, or pilferage; disposal is not allowed. Inventory on hand at the end of a current period is the inventory from the previous period plus any replenishment that arrives less demand in the current period. Replenishments are assumed to be delivered a fixed lead time L periods after being ordered. The time sequence of events within any period is taken to be order, delivery, demand.

We assume no time discounting of costs and postulate an unbounded horizon over which the item is demanded and stocked. We seek to minimize total expected cost per period. The cost of a replenishment quantity is assumed to be linear plus a fixed order cost K. The inventory holding cost is proportional to any end-of-period stock on hand, at unit cost h. The unit penalty cost π is applied to any quantity on backorder at the end-of-period; it is not applied to demands that are equal to or greater than τ (specially handled demands).

We define $\phi(q)$ as the probability distribution of demand. We modify the demand distribution as follows

$$\psi_\tau(q) = \begin{cases} \phi(q) + 1 - \Phi(\tau-1) & q = 0 \\ \phi(q) & 1 \le q < \tau \\ 0 & q \ge \tau \end{cases} \qquad (1)$$

We use the following total expected cost per period model [4] with a given value of τ

$$\frac{K}{1+M(D)} + \sum_{y=s}^{S} r(y) \left[\sum_{q=0}^{y} h(y-q)\, \psi_\tau^{L+1}(q) + \sum_{q=y}^{\infty} \pi(q-y) \psi_\tau^{L+1}(q) \right] \qquad (2)$$

where

$\psi_\tau^{L+1}(x) = L+1^{st}$ fold convolution of $\psi_\tau(x)$,

$$M(x) = \sum_{i=1}^{\infty} \psi_\tau^{i}(x) = \sum_{j=0}^{x} m(j) \qquad (x=0,1,2,\ldots),$$

$$r(S-z) = \frac{\text{Expected number of periods between reorders in which inventory on-hand plus on-order is } S-z \text{ units}}{\text{Expected number of periods between reorders}} = \frac{m(z)}{1+M(D)} \text{ for } 1 \le z \le D,$$

and $D = S-s$.

The expected cost per period function is linear in K, π, and h and we may scale these parameters so that h is unity.

We assume control over replenishment is exercised by an (s,S) replenishment policy: whenever inventory x on-hand and on-order at the start of a period drops below s, an order is placed for a replenishment of size $S-x$.

Given our assumptions, when the demand distribution and the economic parameters are known, there is an optimal policy that has the (s,S) form [3], [5]. We use the Veinott-Wagner algorithm as programmed by [4] to determine optimal (s,S) policies. We modify Kaufman's program to special handle extreme value demands.

It should be noted that (2) does not include any costs for special handling extreme value demands. We determine when cost savings are realized for special handling extreme value demands. If such savings are realized, they would be available to defray the cost of special handling.

1.2 Experimental Design

We study the model for the input parameters shown in Table 1. The demand distributions are Poisson (variance-to-mean ratio of 1:1) and negative binomial (variance-to-mean ratios of 3:1 and 9:1). There are 7 mean values of demand for the Poisson distribution and 8 mean values for the negative binomial distribution. The leadtimes are 0, 1, and 4. The stockout costs are 4, 9, 99, and 199, which result in service levels $(R=\pi/\pi+h)$ of 80%, 90%, 99%, and 99.5% respectively. The set-up cost values are 0, 32, and 64. The cumulative probability values of demand being specially handled $1-\Phi(\tau-1)$ are 0 (no special handling), .05, and .15. These values are approximate since the demand distributions are discrete. We use a full factorial experimental design which generates 2268 combinations of parameters (for mean demand levels .1 and .3 we use only the values 0 and .05 for $1-\Phi(\tau-1)$; see Appendix I). We generate 1440 cases where extreme value demand is special handled and 828 cases without special handling.

2. RESULTS

We examine the reduction in total costs, categorize the cost savings, display the value of demand unfilled versus the reduction in inventory investment, and determine the breakeven special handling cost.

2.1 Reduction in Total Costs

We present the reduction in total costs in three graphs to illustrate the sensitivity to service level R $(\pi/\pi+h)$, leadtime L, and set-up cost K.

SYSTEM PARAMETERS

PARAMETERS	LEVELS	NUMBER OF LEVELS
Variance-to-mean Ratio ($\sigma^2:\mu$)	Poisson ($\sigma^2:\mu=1:1$) Negative Binomial ($\sigma^2:\mu=3:1$ & $9:1$)	3
Mean Demand (μ)* Poisson	.1, .3, 1, 2, 4, 8, 10	7
Negative Binomial	.1, .3, 1, 2, 4, 8, 16, 25	8
Unit Holding Cost (h)	1	1
Replenishment Leadtime (L)	0, 1, 4	3
Unit Penalty Cost (π)	4, 9, 99, 199	4
Replenishment Set-up Costs (K)	0, 32, 64	3
Cumulative Probability of Demand Special Handled ($1-\Phi(\tau-1)$)	0, .05, .15	3

*For mean demand levels .1 and .3, we use only 0 and .05 values for $1 - \Phi(\tau-1)$ (See Appendix I).

TABLE I

Appendix I presents the 54 graphs showing the reduction in total costs for different service levels R. We also show the percent reduction on the graph, where percent reduction is:

$$\frac{\text{Total Expected Cost Without Special Handling} - \text{Total Expected Cost With Special Handling}}{\text{Total Expected Cost Without Special Handling}} \times 100$$

Each figure is for a given set-up cost K, variance-to-mean ratio $\sigma^2:\mu$, leadtime L, and cumulative probability of demand special handled $1-\Phi(\tau-1)$.

In Appendix II we reformat the data presented in Appendix I to display the reduction in total costs for different leadtimes. We show only 12 (of the 72) charts. They are representative, however of the effect of changing leadtimes. We show all combinations for $R=.80$ and $.99$, $1-\Phi(\tau-1)\simeq.15$, $K=0$ and 64, and $\sigma^2:\mu=9:1$, $3:1$, and $1:1$.

Finally we present in Appendix III the reduction in total costs for different set-up costs. Again we present only 12 (of the 72) charts as representative of the effect of changing the set-up cost parameter. We show all combinations for $R=.80$ and $.99$, $1-\Phi(\tau-1)\simeq.15$, $L=1$ and 4, and $\sigma^2:\mu=9:1$, $3:1$, and $1:1$.

The results for the reduction in total costs are fairly consistent over all parameter settings. The percent reduction in costs is monotonically increasing as:

(1) mean demand becomes sufficiently small,

(2) the variance-to-mean ratio becomes sufficiently large,

(3) the service level $(\pi/\pi+h)$ becomes sufficiently large, and

(4) the set-up cost becomes sufficiently small.

2.2 Categorization of Cost Savings

In Appendix IV we categorize the sources of cost savings. Note from (2) that expected costs consist of holding, penalty, and set-up. Appendix IV displays graphically the cost saving (or increase) for each cost category. We show all combinations listed in Table 2. Thus 24 of the 216 possible figures are shown.

CATEGORIZATION OF COST SAVINGS
PARAMETER SETTINGS

PARAMETER	LEVELS	NUMBER OF LEVELS
Variance-to-mean Ratio ($\sigma^2:\mu$)	1:1, 3:1, 9:1	3
Mean Demand (μ)		
Poisson	.1, 2, 8	3
Negative Binomial	.3, 4, 16	3
Unit Holding Cost (h)	1	1
Replenishment Leadtime (L)	1, 4	2
Unit Penalty Cost (π)	4, 99	2
Replenishment Set-up Cost (K)	0, 64	2
Cumulative Probability of Demand Special Handled ($1-\Phi(\tau-1)$		
Poisson	.05	1
Negative Binomial	.15	1

TABLE 2

The results of the categorization of cost savings are nearly consistent over all parameter settings. Special handling of extreme value demands:

(1) decreases the optimal amount to stock, thereby decreasing the amount of holding costs. Since the holding parameter cost equals 1, the reduction in holding costs is equivalent to the reduction in inventory investment (at the end of the period).

(2) decreases the penalty cost incurred. Since the penalty cost is proportional to the size of the demand backlogged, attempting to fill extreme demand generates higher expected penalty costs. Special handling of the extreme value demand thus reduces expected penalty cost.

(3) decreases the frequency of replenishment, thereby decreasing the expected set-up cost. Although the amount being stocked is usually smaller, the quantity demanded is less due to the exclusion of extreme value demands.

2.3 Unfilled Demand Versus the Reduction in Inventory Investment

The motivation for reviewing the unfilled demand versus the reduction in inventory investment derives from the concept of "premium versus protection" in the robust estimation literature [2]. In the special handling of extreme value demand context, the "premium" is the price paid for estimating demand based on a truncated sample. In other words, the premium is the demand left unfilled. The "protection" is the amount saved for the better estimation of ordinary (non-extreme value) demand. In this case, the protection is the reduction in inventory investment. Note we include the extreme value demands that are special handled in the value of unfilled demands. We know from Appendix IV that the demand unfilled not including the extreme value demand is generally smaller in the special handling model than in the standard model. Therefore we compare the savings in inventory investment to the cost of incurring those savings, specifically the increase in the total demand unfilled including extreme value demands.

We compare the difference between the amount of orders left unfilled for the minimum cost policies of the special handling model and the standard model (no special handling) to the difference between the amount of stock held at the end of the period for the minimum cost policies for the special handling model and the standard model. Mathematically the amount of orders left unfilled per period is

$$\sum_{y=s}^{S} r(y) \sum_{q=y}^{\infty} (q-y)\psi_{\tau}^{L+1}(q) + \sum_{q=\tau}^{\infty} q\phi(q) \tag{3}$$

Expected Stock Out Quantity + Expected Demand Special Handled,

where S is the optimal stockage policy and where $\tau=\infty$ is the standard model. The amount of stock held at the end of the period is:

$$\sum_{y=s}^{S} r(y)\left[\sum_{q=0}^{S} (y-q)\psi_{\tau}^{L+1}(q)\right] \tag{4}$$

where $\tau=\infty$ again is the standard model.

Appendix V displays the difference in the value of demand unfilled versus the reduction in inventory investment. We present 24 (of the 276) figures; one for each of the combination of parameters shown in Table 3.

Appendix V shows the amount of inventory investment that can be saved if one is willing to incur an increase in the amount of demand unfilled including specially handled demand. For example, in the case with $\sigma^2:\mu=9:1$, $\mu=4$, $L=0$, $R=.9$, $K=0$, and $1-\Phi(\tau-1)\simeq 15$, 4.2 less units can be stocked at a cost of an increase of 1.1 units of demand left unfilled. As the parameter settings move toward more percent reduction in total expected costs (see section 2.1 above) the difference in the reduction in inventory investment to unfilled demand increases. The point is that under a number of parameter settings ($R>.8$ and $\sigma^2:\mu>1:1$) the model that does not special handle extreme value demand ($1-\Phi(\tau-1)=0$) requires significantly more inventory investment for a relatively small difference in demand left unfilled. Hence the use of a model to special handle extreme value demands can greatly reduce inventory investment, and in a number of practical parameter settings, the benefit from the inventory reduction significantly exceeds the cost incurred from unfilled demand.

UNFILLED DEMAND VERSUS THE REDUCTION IN INVENTORY INVESTMENT PARAMETER SETTINGS

PARAMETER	LEVELS	NUMBER OF LEVELS
Variance-to-mean ratio ($\sigma^2:\mu$)	Poisson ($\sigma^2:\mu$=1:1) Negative Binomial ($\sigma^2:\mu$ = 3:1 & 9:1)	3
Mean Demand (μ)		
Poisson	2, 8	2
Negative Binomial	4, 16	2
Unit Holding Cost (h)	1	1
Replenishment Leadtime (L)	0, 4	2
Unit Penalty Cost (π)	4, 99	2
Replenishment Set-up Cost (K)	0, 32, 64	3
Cumulative Probability of Demand Special Handled ($1-\Phi(\tau-1)$)	.05, .15	2

TABLE 3

2.4 Breakeven Special Handling Cost

In this section, we include in the expected cost per period model (2) a special order cost per period that equates the costs for the special handling model to the costs for the standard model. Thus we solve for K' (fixed special order cost) in the following:

$$\text{Special Order Cost}\begin{pmatrix}\text{Probability of a}\\ \text{Special Order}\\ \text{in a Period}\end{pmatrix} = \begin{matrix}\text{Total Cost}\\ \text{Per Period}\\ \text{for the}\\ \text{Standard Model}\end{matrix} - \begin{matrix}\text{Total Cost}\\ \text{Per Period}\\ \text{for the}\\ \text{Special Handling}\\ \text{Model}\end{matrix}, \tag{5}$$

which is equivalent to

$$K' \sum_{q=\tau}^{\infty} \phi(q) = \text{Cost Savings per Period} .$$

Table 4 provides the maximum cost for special handling extreme value demands where special handling still reduces total expected costs per period. For example for $\sigma^2:\mu=9:1$, $\mu=4$, $1-\Phi(\tau-1)\approx.05$, $L=4$, $K=32$, and $R=.9$, the special handling model will reduce costs as long as the special handling costs is less than \$238, which is almost 7.5 times larger than the ordinary ordering cost (K). We show the breakeven special handling cost for $\mu=.3$ and $\mu=4$, $1-\Phi(\tau-1)=.05$, $L=0$ and 4, and all other parameter settings.

TABLE 4

BREAKEVEN SPECIAL HANDLING COSTS

$\mu = .3$

L=0 $\sigma^2:\mu$	9:1				3:1				1:1			
R	.8	.9	.99	.995	.8	.9	.99	.995	.8	.9	.99	.995
K=0	22	50	250	336	13	21	81	101	4	9	36	46
K=32	50	73	257	342	30	37	85	105	17	18	39	46
K=64	73	92	265	356	41	47	91	111	21	25	40	52
L=4												
K=0	94	162	448	545	37	60	123	140	16	18	32	47
K=32	111	175	462	552	47	64	130	151	20	25	45	53
K=64	123	187	470	562	55	70	135	158	26	33	48	56

$\mu = 4$

L=0												
K=0	62	109	364	461	24	42	138	174	8	16	54	66
K=32	73	113	318	399	33	43	103	128	16	19	38	46
K=64	85	120	312	390	41	51	106	130	23	25	39	49
L=4												
K=0	164	239	510	606	55	79	155	360	13	20	49	63
K=32	166	238	498	584	59	80	148	164	20	26	47	57
K=64	171	241	496	580	64	84	150	165	26	31	51	58

Table 5 shows the frequency distribution for breakeven special handling costs. The table shows the percentage of cases where the breakeven special handling cost is: less than the normal set-up cost, between the normal set-up cost and 1.5 of the normal set-up cost, etc.. We present all cases with a positive set-up cost (K=32 and 64). We use $1-\Phi(\tau-1)\approx.05$ which is more favorable than $1-\Phi(\tau-1)\approx.15$. In 71.9% of the cases breakeven special handling costs exceeded the normal set-up cost. Excluding the Poisson cases $(\sigma^2:\mu=1:1)$, in 89.8% of the cases breakeven special handling costs exceeded the normal set-up cost, and in nearly half of those cases, breakeven special handling costs more than doubled the normal set-up cost.

SPECIAL HANDLING COST
FREQUENCY DISTRIBUTION

$1-\Phi(\tau-1)=.05$	Variance-to-Mean Ratio			
	9:1	3:1	1:1	Total
$0 \le K' < K$	.0	20.3	69.0	28.1
$K \le K' < 1.5K$	2.6	16.7	19.0	12.5
$1.5K \le K' < 2K$	6.3	18.2	7.7	10.9
$2K \le K' < 4K$	23.4	26.6	3.0	18.3
$4K \le K' < 10K$	38.0	18.2	1.2	19.9
$K' \le 10K$	29.7	.0	.0	10.3

TABLE 5

When special handling costs are fixed, the higher the cost reduction (as opposed to the higher the percent cost reduction) the more favorable the special handling model. Hence breakeven special handling costs monotonically increase as

1. the service level increases,
2. the leadtime increases, and
3. the variance-to-mean ratio increases.

3. SUMMARY

Table 6 presents the frequency distribution for percent cost reduction categorized for each variance-to-mean ratio and for the total of all cases. In 99.5% (only 5 cases with $\sigma^2:\mu=1:1$ and 4 cases with $\sigma^2:\mu=3:1$ were costs not reduced) of the 1440 total cases we examined (Table 1) a periodic review inventory model that special handles extreme value demand reduces total expected costs (excluding special handling costs). Excluding $\sigma^2:\mu=1:1$, 50% of the remaining cases reduced total cost by at least one half.

TABLE 6

PERCENT COST REDUCTION
FREQUENCY DISTRIBUTION

	Variance-to-Mean Ratio			
Percent Cost Savings	9:1	3:1	1:1	Total
Less than 0	.0	.4	1.2	.5
[0- 10)	.2	8.3	39.4	14.8
[10- 20)	7.9	22.0	34.0	20.7
[20- 30)	13.1	19.4	15.0	15.9
[30- 40)	14.7	14.3	4.2	11.4
[40- 50)	12.7	10.1	2.8	8.8
[50- 60)	12.1	10.7	2.1	8.6
[60- 70)	10.5	8.1	1.4	6.9
[70- 80)	10.3	4.6	.0	5.2
[80- 90)	12.1	2.0	.0	4.9
[90-100)	6.3	.0	.0	2.2

Most of the cost reduction is as a result of decreased inventory investment, with a smaller proportion due to decreased penalty costs and decreased set-up costs. In instances where the percent total cost reduction is large, the amount saved in inventory investment is significantly larger than the total amount of demand that is left unfilled. Hence special handling can greatly reduce inventory investment without significantly increasing the penalty cost for not filling demand.

REFERENCES

1. Blazer, Douglas and Marilyn McClelland, Technical Report #19; "An Inventory Model for Special Handling Extreme Value Demands", School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, December 1981.

2. Huber, P. J., "Robust Statistics; A Review (The 1972 Wald Lecture)", *Annals of Mathematics and Statistics*, 43, 1041-1067.

3. Iglehart, D., "Optimality of (s,S) Policies in the Infinite Horizon Dynamic Inventory Problem", *Management Science*, Vol. 9, 1963.

4. Kaufman, R., "Computer Programs for (s,S) Policies under Independent or Filtered Demands", ONR and ARO Technical Report 5, School of Organization and Management, Yale University, 1976.

5. Veinott, A. F. and H. M. Wagner, "Computing Optimal (s,S) Inventory Policies", *Management Science*, Vol. 11, 1965.

APPENDIX I

REDUCTION IN TOTAL COSTS FOR DIFFERENT SERVICE LEVELS

Table I-1 provides the value of and the corresponding cumulative probability or demand special handled for the experimental design shown in Table 1. Since the distributions are discrete the values of τ were chosen so that the cumulative probability of demand special handled is approximately equal to .05 and .15. There are some minor inconsistencies in the data (Appendices I-V) due to the discrete nature of the distributions. These inconsistencies are especially apparent for the Poisson distribution because the differences to .05 and .15 are larger and the relatively poorer performance of the special handling model with the Poisson distribution (variance-to-mean ratio of 1:1).

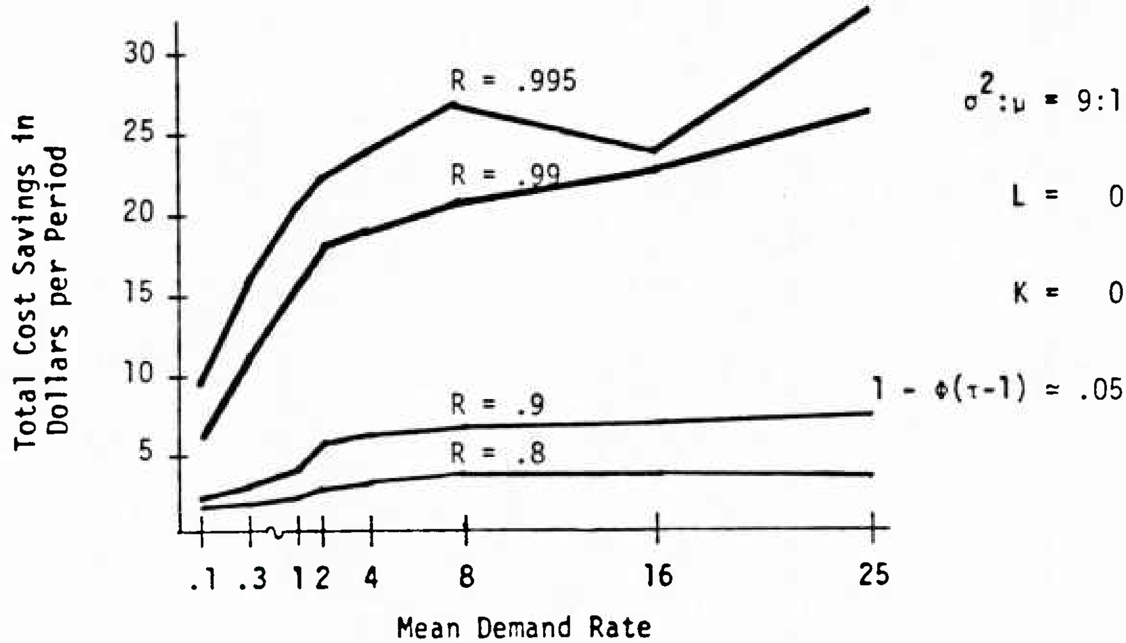
TABLE I-1

VALUES OF CUMULATIVE PROBABILITY OF DEMAND SPECIAL HANDLED

$\sigma^2:\mu$	9:1		3:1		1:1	
Mean Demand (μ)	τ	$1-\Phi(\tau-1)$	τ	$1-\Phi(\tau-1)$	τ	$1-\Phi(\tau-1)$
.1	2	.0163	2	.0219	2	.0047
.3	2	.0484	2	.0671	2	.0368
1	6	.0550	5	.0493	4	.0190
	2	.1557	3	.1340	3	.0803
2	10	.0557	7	.0585	5	.0527
	5	.1412	5	.1317	4	.1429
4	16	.0540	11	.0540	8	.0511
	9	.1510	8	.1431	7	.1107
8	25	.0526	17	.0604	13	.0638
	16	.1519	13	.1659	12	.1119
10	--	--	--	--	16	.0487
	--	--	--	--	14	.1355
16	40	.0490	29	.0514	--	--
	28	.1519	23	.1668	--	--
25	54	.0500	40	.0599	--	--
	40	.1552	34	.1596	--	--

APPENDIX I

Reduction in Total Costs for Different Service Levels



I - 2

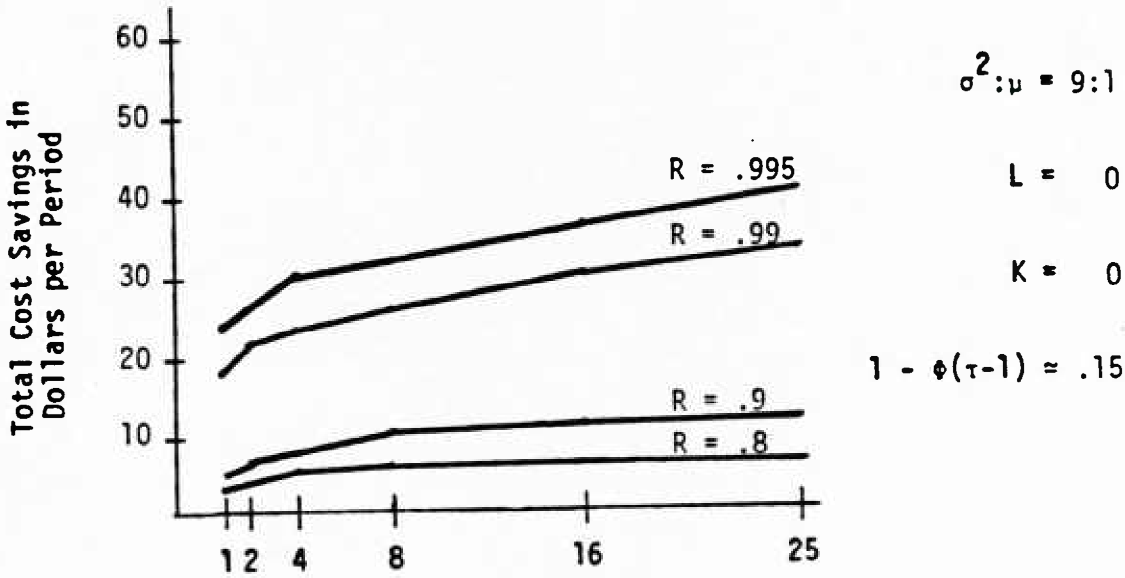
Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	.36/89.2	.80/89.2	6.43/86.7	9.93/91.6
.3	1.08/89.8	2.42/89.8	12.11/92.6	16.24/94.4
1	2.24/58.9	4.25/61.6	15.94/77.5	20.60/81.6
2	2.88/45.8	5.21/50.7	17.80/69.4	22.82/75.5
4	3.34/35.0	5.89/40.9	19.63/62.1	24.91/67.4
8	3.75/27.5	6.60/33.8	21.74/55.6	27.32/60.7
16	4.00/21.0	7.15/27.2	23.13/47.2	23.32/50.9
25	4.29/18.3	7.89/24.8	26.14/45.7	32.42/50.3

APPENDIX I

Reduction in Total Costs for Different Service Levels



I - 3

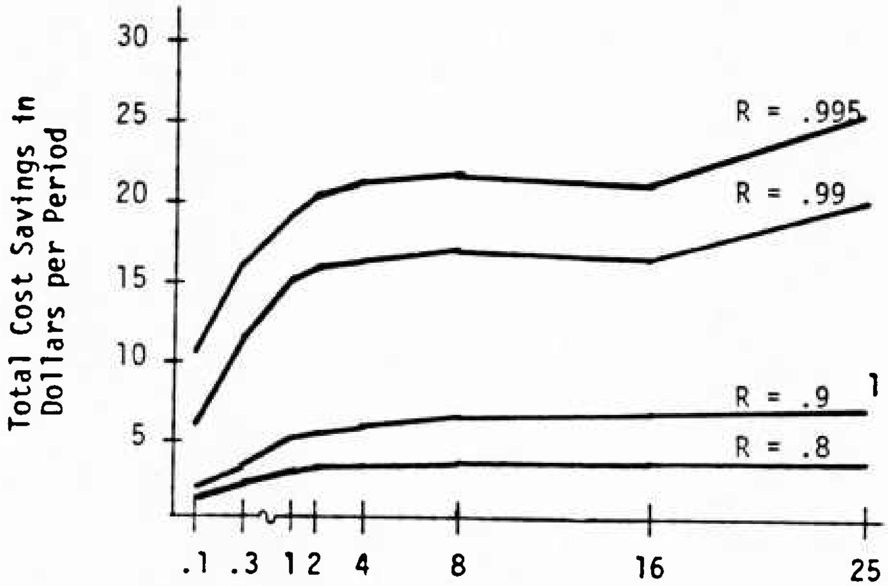
Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	3.46/91.1	6.15/89.0	19.66/95.6	24.35/96.4
2	4.50/71.5	7.67/74.6	22.23/86.6	27.23/88.9
4	5.65/59.3	9.30/64.6	25.24/79.9	30.59/82.7
8	6.57/48.1	10.74/55.0	28.48/72.8	34.39/76.5
16	7.22/37.9	12.05/45.9	31.94/65.2	38.28/68.9
25	7.44/31.7	12.91/40.6	35.20/61.6	42.48/65.9

APPENDIX I

Reduction in Total Costs for Different Service Levels



$\sigma^2 : \mu = 9:1$

$L = 0$

$K = 32$

$1 - \Phi(\tau - 1) \simeq .05$

I - 4

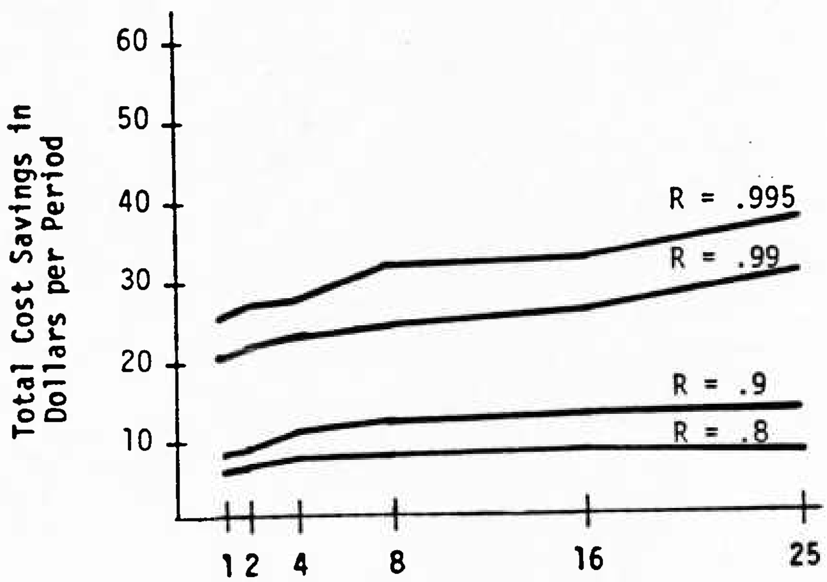
Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	.88/69.3	1.32/75.0	6.83/85.0	10.11/88.3
.3	2.42/70.0	3.53/76.0	12.44/86.4	16.53/89.4
1	3.26/40.8	5.04/48.4	15.50/65.7	19.67/69.7
2	3.65/30.4	5.68/36.7	16.55/54.3	20.81/58.8
4	3.96/22.6	6.09/27.4	17.16/43.8	21.55/48.4
8	4.23/16.8	6.45/20.7	17.95/35.4	22.50/39.8
16	4.28/12.0	6.59/15.2	17.86/26.9	21.81/29.8
25	4.54/10.2	7.10/13.3	20.40/25.6	25.88/29.8

APPENDIX I

Reduction in Total Costs for Different Service Levels



$\sigma^2 : \mu = 9:1$

$L = 0$

$K = 32$

$1 - \Phi(\tau - 1) = .15$

I - 5

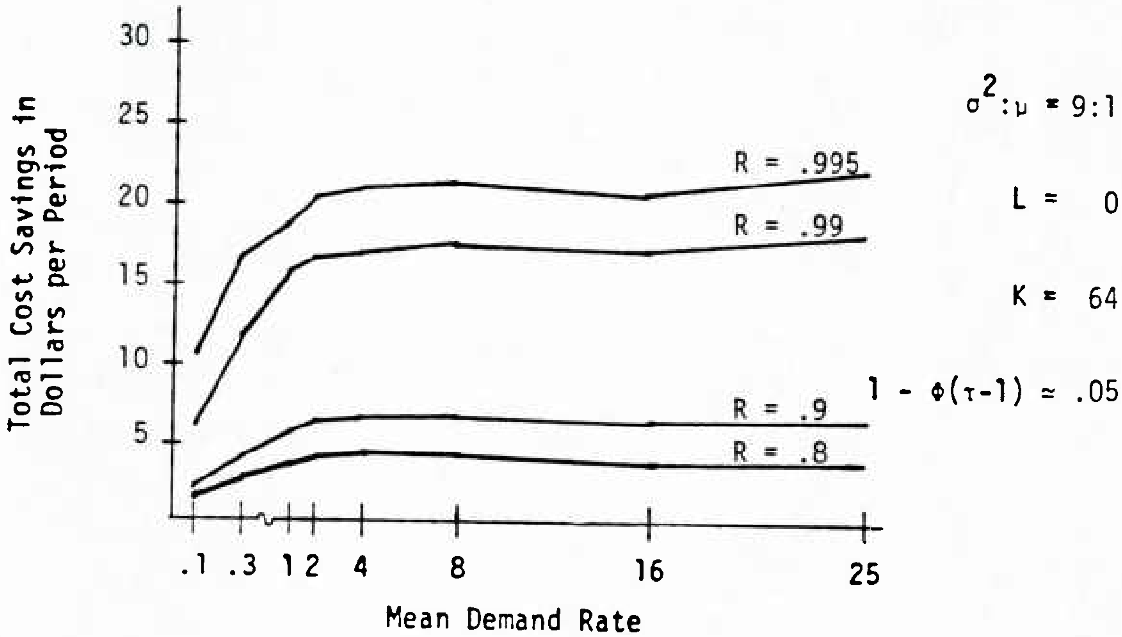
Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	6.04/75.5	8.31/79.8	20.84/88.3	25.48/90.2
2	6.42/53.5	9.19/59.3	22.10/72.5	26.61/75.2
4	7.47/42.6	10.56/47.6	24.14/61.6	28.96/65.1
8	8.29/32.8	11.50/36.9	25.90/51.1	31.04/54.9
16	8.73/24.4	12.15/28.1	27.62/41.6	32.97/45.1
25	9.03/20.3	12.81/24.0	31.50/39.6	38.37/44.1

APPENDIX I

Reduction in Total Costs for Different Service Levels



I - 6

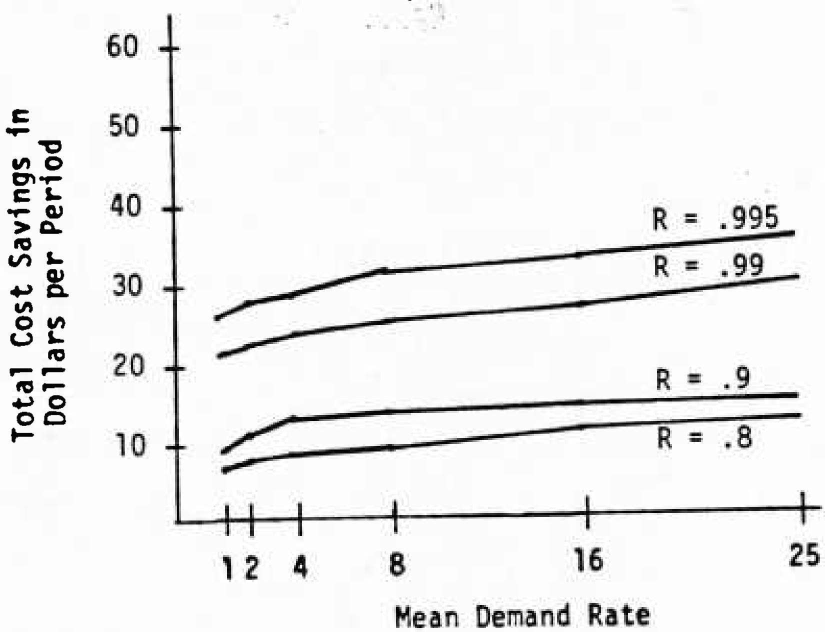
Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	1.40/65.6	1.85/70.1	7.10/83.8	10.19/85.8
.3	3.55/69.9	4.45/73.5	12.85/84.0	16.92/87.3
1	4.14/38.9	5.73/44.7	15.70/61.3	19.68/65.1
2	4.47/28.5	6.28/33.3	16.45/49.0	20.61/53.5
4	4.60/20.3	6.50/24.0	16.87/38.4	21.04/42.8
8	4.76/14.7	6.69/17.5	17.38/30.1	21.76/34.2
16	4.67/10.2	6.61/12.3	16.96/22.0	20.66/24.7
25	4.77/ 8.3	6.87/10.3	18.50/19.8	23.06/22.9

APPENDIX I

Reduction in Total Costs for Different Service Levels



$\sigma^2:\mu = 9:1$

L = 0

K = 64

$1 - \Phi(\tau-1) = .15$

I - 7

Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	7.77/73.1	9.85/76.8	21.89/85.5	26.49/87.7
2	7.99/50.9	10.48/55.5	22.86/68.1	27.40/71.2
4	8.99/39.7	11.83/43.7	24.83/56.6	29.50/59.9
8	9.79/30.2	12.71/33.2	26.60/46.0	31.55/49.5
16	10.42/22.7	13.65/25.4	28.54/37.0	33.78/40.3
25	10.80/18.8	14.32/21.5	30.71/32.9	36.64/36.4

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 9:1$ $\quad L = 1$ $\quad K = 0$ $\quad 1 - \Phi(\tau-1) \simeq .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	0.71/89.2	1.61/89.2	9.86/90.9	13.74/93.2
.3	2.15/89.8	4.32/88.7	16.20/94.0	20.55/94.8
1	3.86/61.3	6.83/66.4	19.90/77.6	24.01/78.4
2	4.74/49.7	7.94/55.2	20.97/66.3	25.16/68.0
4	5.38/39.4	8.79/45.0	21.60/55.3	26.06/57.9
8	5.94/31.2	9.41/35.9	22.42/45.8	26.98/48.5
16	6.07/23.0	9.49/26.8	21.51/34.4	24.37/34.7
25	6.25/19.3	9.97/23.2	24.05/32.5	28.27/34.3

$\sigma^2:\mu = 9:1$ $\quad L = 1$ $\quad K = 0$ $\quad 1 - \Phi(\tau-1) \simeq .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	5.62/89.3	9.38/91.2	24.12/94.0	28.80/94.0
2	7.05/73.9	11.11/77.2	26.20/82.9	31.07/84.2
4	8.64/63.3	13.13/67.2	28.59/73.1	33.55/74.6
8	9.81/51.5	14.42/54.9	31.13/63.6	36.68/66.0
16	10.16/38.6	15.09/42.7	33.31/53.3	39.09/55.7
25	9.80/30.2	15.21/35.4	35.86/48.5	42.86/51.9

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 9:1$ $L = 1$ $K = 32$ $1 - \Phi(\tau-1) = .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	1.24/74.1	2.13/79.8	10.08/88.3	13.95/91.2
.3	3.29/75.2	5.16/80.6	16.52/89.3	20.89/91.2
1	4.62/47.6	7.28/54.1	19.57/68.4	23.99/71.2
2	5.28/36.4	8.09/42.1	20.50/56.5	24.73/59.4
4	5.68/27.1	8.56/31.9	20.80/44.8	25.04/47.9
8	6.00/20.1	8.93/24.0	21.00/34.9	25.21/37.7
16	5.97/14.3	8.87/17.3	19.38/24.5	21.59/24.8
25	6.15/11.9	9.27/14.7	21.41/22.5	24.87/23.9

I - 10

$\sigma^2:\mu = 9:1$ $L = 1$ $K = 32$ $1 - \Phi(\tau-1) = .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	7.69/79.2	11.13/82.8	25.65/89.7	30.39/90.6
2	8.57/59.1	12.27/63.8	27.03/74.5	31.77/76.2
4	9.97/47.5	13.99/52.1	28.91/62.3	33.54/64.1
8	10.97/36.9	15.21/41.0	30.66/51.0	35.74/53.5
16	11.53/27.6	15.89/31.0	31.90/40.3	37.01/42.6
25	11.47/22.1	16.02/25.4	33.85/35.5	40.08/38.5

I - 11

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 9:1$ $L = 1$ $K = 64$ $1 - \Phi(\tau-1) \approx .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	1.76/69.3	2.49/73.8	10.16/85.8	14.01/89.2
.3	4.21/72.8	5.83/77.1	16.91/87.3	21.26/89.4
1	5.31/43.8	7.79/50.0	19.77/64.7	23.94/67.5
2	5.81/32.8	8.48/38.0	20.59/52.4	24.81/55.6
4	6.11/23.9	8.80/28.0	20.73/40.7	24.90/43.8
8	6.33/17.5	9.09/20.7	20.77/31.0	24.85/33.7
16	6.25/12.2	8.91/14.6	18.75/21.0	20.75/21.3
25	6.41/10.1	9.21/12.2	20.47/18.9	23.65/20.2

I - 12

$\sigma^2:\mu = 9:1$ $L = 1$ $K = 64$ $1 - \Phi(\tau-1) \approx .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	9.20/76.0	12.45/80.0	26.70/87.4	31.48/88.7
2	9.73/54.9	13.34/59.7	27.85/70.8	32.54/72.9
4	11.18/43.8	15.05/47.9	29.71/58.3	34.33/60.4
8	12.18/33.5	16.26/37.1	31.47/47.0	36.47/49.5
16	12.89/25.2	17.11/28.0	32.87/36.8	37.93/39.0
25	13.12/20.6	17.46/23.1	34.91/32.2	41.00/35.0

I - 13

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 9:1$ L = 4 K = 0 $1 - \Phi(\tau-1) \approx .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	1.78/89.2	3.73/88.5	15.04/93.4	19.29/94.2
.3	4.56/88.1	7.86/89.3	21.69/92.5	26.39/93.2
1	7.16/66.5	11.18/70.2	25.40/75.1	29.80/75.7
2	8.24/54.1	12.28/57.1	26.78/63.8	31.57/65.7
4	8.86/41.9	12.92/44.7	27.56/52.1	32.71/54.7
8	9.34/31.9	13.44/34.5	27.88/41.1	32.76/43.1
16	9.15/22.6	13.04/24.6	22.7/25.5	21.80/22.1
25	9.15/18.3	13.35/20.6	25.70/24.0	27.08/22.9

I - 14

$\sigma^2:\mu = 9:1$ L = 4 K = 0 $1 - \Phi(\tau-1) \approx .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	9.86/91.6	14.69/92.3	31.73/93.8	36.76/93.4
2	11.59/76.2	16.78/78.0	34.23/81.6	39.76/82.7
4	13.60/64.3	19.15/66.3	36.99/70.0	42.08/70.4
8	15.02/51.4	20.86/53.5	39.93/58.8	45.76/60.2
16	15.31/37.9	21.35/40.3	40.73/45.8	45.21/46.8
25	14.32/28.7	20.56/31.7	42.32/39.6	59.71/50.5

I - 15

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 9:1$ $L = 4$ $K = 32$ $1 - \Phi(\tau-1) \approx .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	2.31/80.4	4.06/83.0	15.23/91.5	19.47/92.7
.3	5.39/81.2	8.49/83.7	22.36/90.6	26.70/90.6
1	7.59/55.4	11.36/60.4	25.36/69.2	29.67/70.4
2	8.45/43.0	12.33/47.5	26.52/57.1	31.12/59.2
4	8.96/32.3	12.84/36.1	26.90/45.0	31.51/47.2
8	9.35/24.1	13.21/27.0	27.08/34.6	31.65/36.6
16	9.11/16.8	12.70/18.8	21.49/20.6	20.30/17.7
25	9.16/13.6	13.00/15.7	24.28/19.2	25.28/18.3

$\sigma^2:\mu = 9:1$ $L = 4$ $K = 32$ $1 - \Phi(\tau-1) \approx .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	11.41/83.2	16.10/85.6	33.02/90.8	38.35/90.9
2	12.77/65.0	17.77/68.5	34.96/75.3	40.26/76.6
4	14.73/53.2	20.09/56.4	37.53/62.8	42.43/63.6
8	16.13/41.6	21.77/44.5	40.26/51.4	45.86/53.0
16	16.68/30.7	22.42/33.0	40.93/39.2	46.29/40.4
25	16.10/23.9	21.92/26.4	42.42/33.5	48.79/35.3

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 9:1$ $L = 4$ $K = 64$ $1 - \Phi(\tau-1) \approx .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	2.59/74.1	4.32/80.2	15.27/89.7	19.50/91.2
.3	5.96/77.5	9.05/81.6	22.73/89.0	27.22/89.7
1	8.02/51.3	11.68/56.4	25.53/66.3	29.82/67.7
2	8.82/39.3	12.61/43.8	26.55/53.8	31.09/56.1
4	9.24/29.1	13.03/32.8	26.80/41.8	31.33/44.1
8	9.58/21.4	13.34/24.3	26.85/31.7	31.29/33.6
16	9.30/14.8	12.78/16.8	21.17/18.6	19.78/15.9
25	9.41/12.1	13.10/13.9	23.80/17.2	24.53/16.3

I - 18

$\sigma^2:\mu = 9:1$ $L = 4$ $K = 64$ $1 - \Phi(\tau-1) \approx .15$

Service Levels

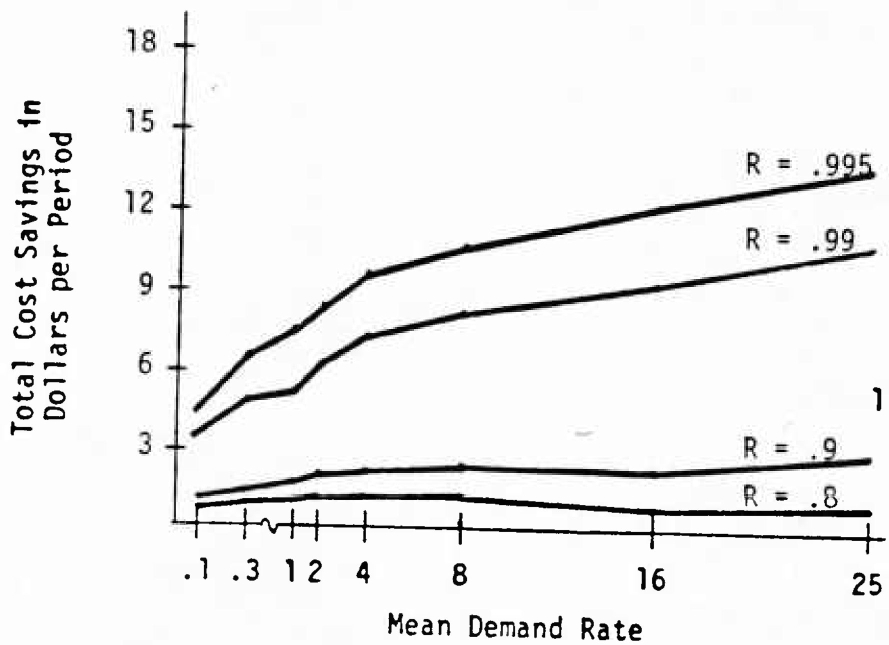
Mean Demand	R = .8	R = .9	R = .99	R = .995
1	12.59/80.5	17.11/82.6	33.97/88.2	39.36/89.4
2	13.73/61.1	18.70/64.9	35.79/72.6	41.10/74.2
4	15.68/49.3	20.99/52.8	38.28/59.7	43.13/60.7
8	17.10/38.3	22.68/41.3	40.98/48.3	46.53/50.0
16	17.78/28.4	23.47/30.8	41.91/36.8	46.28/37.3
25	17.50/22.5	23.27/24.7	43.26/31.2	49.69/33.0

I - 19

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels



$\sigma^2 : \mu = 3:1$

$L = 0$

$K = 0$

$1 - \Phi(\tau - 1) \simeq .05$

I - 20

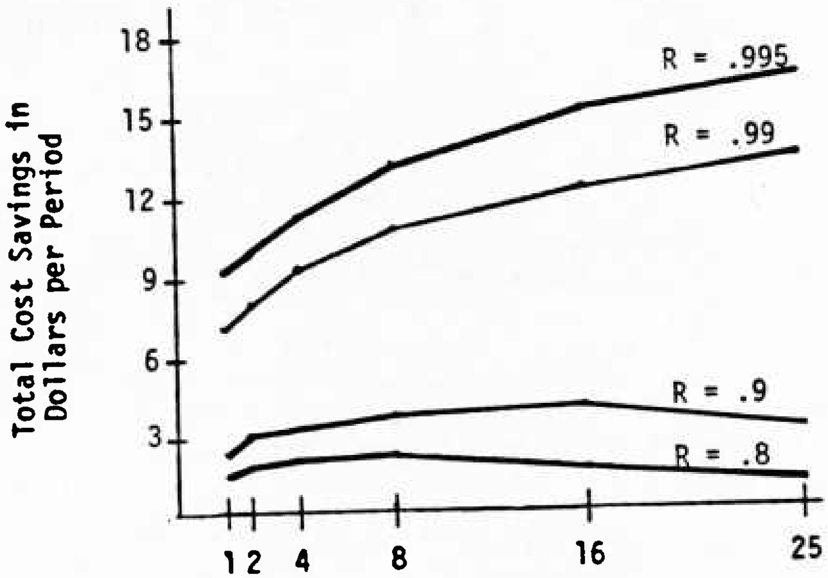
Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	0.27/68.4	.62/68.4	3.31/77.2	4.51/82.2
.3	0.86/71.7	1.42/65.0	5.42/85.6	6.77/88.2
1	0.92/33.7	1.64/39.8	5.81/63.5	7.40/68.9
2	1.23/31.0	2.08/37.0	6.79/60.0	8.54/65.4
4	1.28/23.0	2.27/29.9	7.46/52.6	9.37/58.2
8	1.48/19.4	2.72/26.6	8.89/49.1	11.12/54.7
16	1.44/13.7	2.84/20.5	9.89/41.9	12.57/47.9
25	1.44/11.1	3.17/18.7	11.53/40.9	14.57/46.6

APPENDIX I

Reduction in Total Costs for Different Service Levels



$\sigma^2 : \mu = 3:1$

$L = 0$

$K = 0$

$1 - \Phi(\tau - 1) = .15$

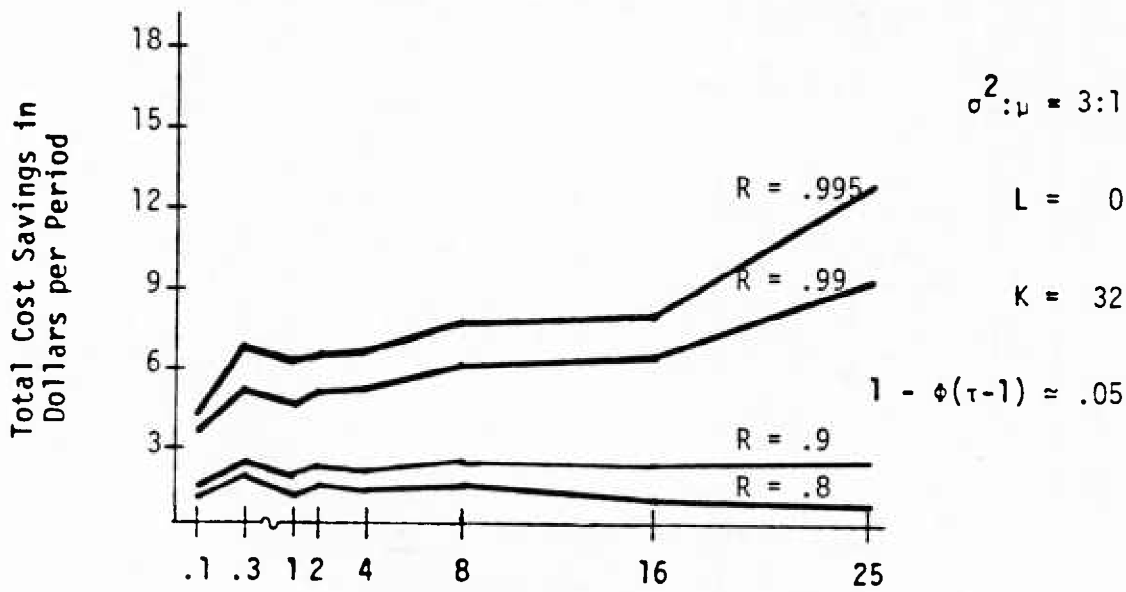
Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	1.64/59.9	2.55/61.8	7.52/82.3	9.12/84.9
2	1.89/47.7	3.05/54.2	8.40/74.3	10.08/77.2
4	2.14/38.8	3.57/47.0	9.70/68.3	11.61/72.1
8	2.42/31.8	4.14/40.4	11.37/62.8	13.58/66.8
16	2.13/20.3	4.27/30.8	12.85/54.4	15.39/58.6
25	1.57/12.1	4.08/24.1	13.93/49.4	16.97/54.3

APPENDIX I

Reduction in Total Costs for Different Service Levels



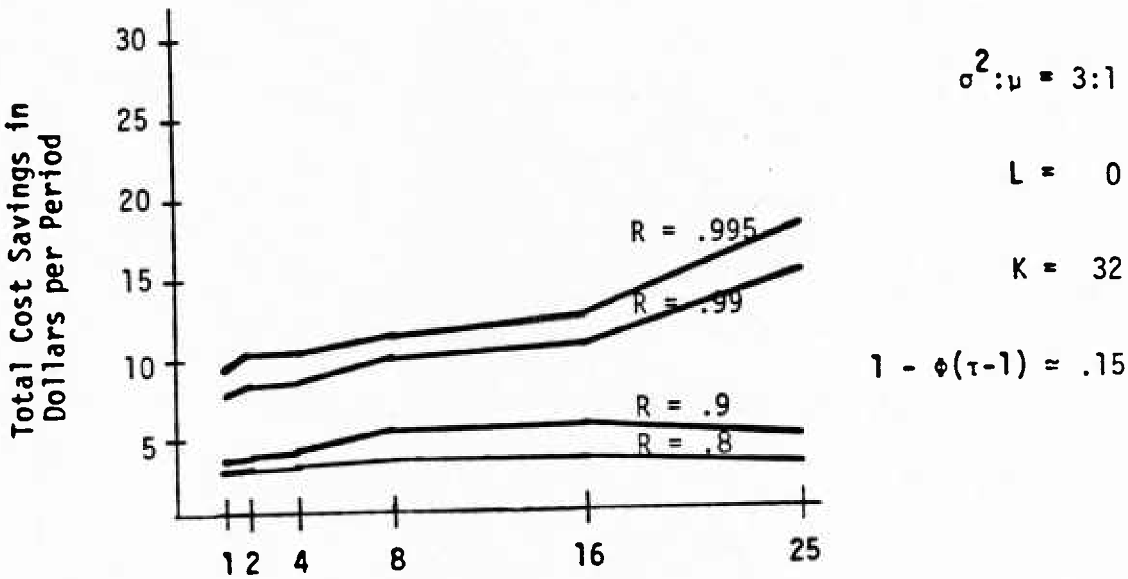
Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	0.88/45.7	1.14/50.1	3.32/62.6	4.51/69.5
.3	2.02/50.7	2.50/54.3	5.68/67.2	7.05/71.8
1	1.57/20.4	2.11/23.5	4.99/36.2	6.13/39.9
2	1.81/16.5	2.42/19.0	5.43/29.6	6.85/34.1
4	1.78/11.4	2.34/13.1	5.58/22.5	6.93/25.9
8	1.98/ 9.0	2.68/10.6	6.44/19.1	8.02/22.3
16	1.75/ 5.6	2.47/ 7.0	6.63/14.4	8.36/17.1
25	1.71/ 4.4	2.60/ 5.9	9.93/17.5	13.23/22.0

APPENDIX I

Reduction in Total Costs for Different Service Levels



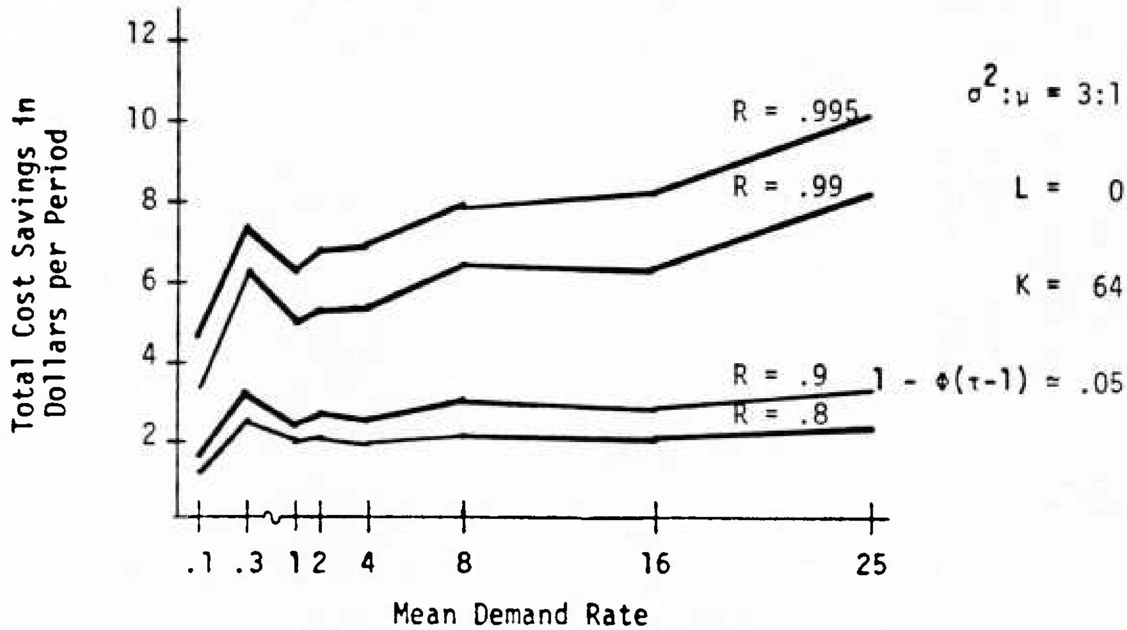
Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	3.19/41.3	4.01/44.7	7.71/55.9	9.28/60.4
2	3.22/29.3	4.07/32.0	7.92/43.1	10.17/48.7
4	3.66/23.4	4.54/25.4	8.81/35.6	10.37/38.8
8	4.27/19.3	5.32/21.1	10.34/30.7	12.20/33.9
16	4.32/13.9	5.61/15.9	11.44/24.8	13.44/27.4
25	3.73/ 9.6	5.10/11.7	15.49/27.3	18.81/31.4

APPENDIX I

Reduction in Total Costs for Different Service Levels



I - 24

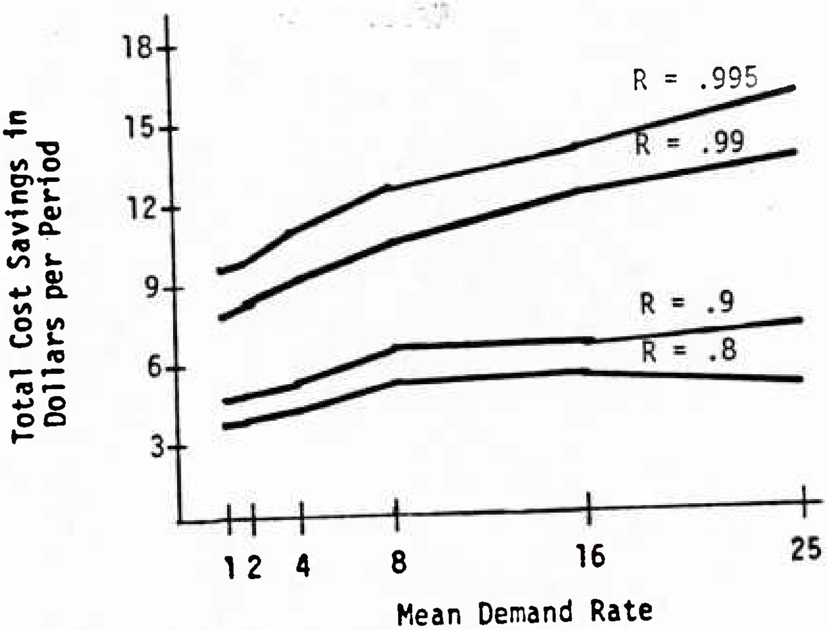
Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	1.40/47.3	1.57/48.9	3.50/58.5	4.68/65.3
.3	2.74/48.9	3.16/51.4	6.12/62.2	7.42/66.6
1	2.05/19.5	2.49/21.3	5.13/31.0	6.29/34.9
2	2.31/15.4	2.80/16.8	5.72/25.6	6.92/28.8
4	2.23/10.5	2.73/11.6	5.73/18.8	7.01/21.6
8	2.52/ 8.4	3.11/ 9.3	6.54/15.6	8.00/18.0
16	2.27/ 5.4	2.86/ 6.1	6.48/11.2	8.18/13.4
25	2.60/ 4.9	3.49/ 6.0	8.15/11.3	10.06/13.4

APPENDIX I

Reduction in Total Costs for Different Service Levels



$\sigma^2:\mu = 3:1$

L = 0

K = 64

$1 - \Phi(\tau-1) \approx .15$

I - 25

Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	4.16/39.6	4.90/41.9	8.39/50.8	9.86/54.8
2	4.21/28.1	4.99/29.9	8.66/38.7	9.91/41.2
4	4.73/22.3	5.56/23.6	9.52/31.2	11.13/34.2
8	5.70/19.0	6.61/19.9	11.25/26.8	12.97/29.2
16	5.93/14.0	6.87/14.7	12.15/20.9	14.05/23.0
25	5.78/10.9	7.56/12.4	14.03/19.5	16.43/21.8

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 3:1 \quad L = 1 \quad K = 0 \quad 1 - \Phi(\tau-1) \approx .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	0.55/68.4	1.19/67.8	4.46/81.0	5.67/83.1
.3	1.32/66.0	2.32/72.0	6.31/80.3	7.49/80.4
1	1.51/38.1	2.44/43.3	6.12/54.1	7.34/56.2
2	1.90/34.5	2.97/39.0	6.83/48.1	8.28/51.4
4	1.96/25.8	3.03/29.6	7.11/39.3	8.51/41.9
8	2.12/20.1	3.35/31.8	8.29/35.2	10.11/38.5
16	1.80/12.4	3.17/16.7	8.86/28.3	10.92/31.6
25	1.46/ 8.1	3.11/13.4	9.89/26.2	12.41/29.8

$\sigma^2:\mu = 3:1 \quad L = 1 \quad K = 0 \quad 1 - \Phi(\tau-1) \approx .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	2.45/61.9	3.85/68.3	8.14/72.0	9.76/74.7
2	2.80/50.8	4.08/53.6	8.94/63.0	10.45/64.8
4	3.08/40.5	4.55/44.5	9.82/54.3	11.74/57.8
8	3.15/29.9	4.80/34.6	11.33/48.1	13.50/51.4
16	2.09/14.3	4.09/21.6	11.63/37.2	13.90/67.3
25	0.64/ 3.5	2.94/12.6	12.50/33.1	16.20/38.9

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 3:1$ $L = 1$ $K = 32$ $1 - \Phi(\tau-1) \simeq .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	1.07/49.4	1.56/55.3	4.50/69.2	5.75/73.6
.3	2.35/53.6	3.19/57.9	7.00/70.2	8.20/72.0
1	1.97/23.4	2.66/26.3	6.06/38.3	7.19/41.0
2	2.28/19.0	3.08/21.7	6.60/31.4	7.81/34.1
4	2.23/13.1	3.04/15.3	6.43/22.8	7.56/24.8
8	2.46/10.3	3.39/12.2	7.41/19.3	8.96/21.7
16	2.10/ 6.2	3.22/ 8.2	7.31/13.9	8.91/15.8
25	1.80/ 4.3	2.85/ 5.9	8.15/12.6	10.29/14.9

I - 28

$\sigma^2:\mu = 3:1$ $L = 1$ $K = 32$ $1 - \Phi(\tau-1) \simeq .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	3.74/44.3	4.79/47.3	8.95/56.6	10.47/59.7
2	3.85/32.1	5.01/35.2	9.33/44.4	10.76/46.9
4	4.30/25.3	5.55/27.9	10.12/35.9	11.60/38.0
8	4.93/20.6	6.27/22.5	11.60/30.2	13.44/32.6
16	4.46/13.2	5.86/15.0	11.79/22.4	13.57/24.1
25	3.15/ 8.1	4.61/ 9.5	12.66/19.6	16.67/24.2

I - 29

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 3:1$ $L = 1$ $K = 64$ $1 - \Phi(\tau-1) \approx .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	1.51/48.4	1.86/51.2	4.68/65.1	5.92/69.8
.3	3.00/50.8	3.70/54.2	7.36/65.6	8.68/68.3
1	2.32/21.0	6.12/54.1	6.24/33.8	7.28/36.1
2	2.67/17.0	3.43/19.0	6.77/27.2	7.97/29.7
4	2.58/11.6	3.32/13.1	6.58/19.4	7.60/21.0
8	2.90/ 9.2	3.76/10.5	7.58/16.3	9.11/18.4
16	2.50/ 5.6	3.32/ 6.6	7.33/11.4	8.82/13.0
25	2.49/ 4.5	3.52/ 5.6	8.79/11.0	10.90/13.0

I - 30

$\sigma^2:\mu = 3:1$ $L = 1$ $K = 64$ $1 - \Phi(\tau-1) \approx .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	4.55/41.2	5.60/43.8	9.79/53.1	11.09/55.1
2	4.71/29.9	5.76/32.0	10.04/40.4	11.35/42.3
4	5.25/23.6	6.44/25.4	10.88/32.1	12.32/34.1
8	6.20/19.7	7.47/20.9	12.59/27.1	14.50/29.3
16	6.07/13.7	5.86/15.0	12.59/19.5	14.26/21.0
25	5.06/ 9.1	6.43/10.3	14.34/18.0	17.61/21.0

I - 31

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 3:1$ $L = 4$ $K = 0$ $1 - \Phi(\tau-1) = .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	1.17/64.9	2.02/68.3	5.58/75.3	6.91/77.9
.3	2.51/73.5	4.02/80.9	8.28/79.7	9.41/78.5
1	2.51/41.2	3.62/43.4	7.74/50.5	9.02/52.0
2	2.91/34.5	4.19/37.2	8.61/43.8	10.09/45.8
4	2.96/25.3	4.24/27.7	8.36/32.5	19.44/50.3
8	3.10/19.2	4.55/21.6	9.78/28.5	11.77/31.6
16	2.32/10.3	3.78/13.0	9.15/19.7	10.86/21.3
25	1.09/ 3.9	2.75/ 7.7	9.68/17.1	12.60/20.3

I - 32

$\sigma^2:\mu = 3:1$ $L = 4$ $K = 0$ $1 - \Phi(\tau-1) = .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	3.93/64.3	5.52/66.2	10.72/70.0	12.22/70.5
2	4.27/50.6	5.96/52.9	11.34/57.7	12.99/59.0
4	4.66/39.9	6.49/42.4	12.26/47.6	23.87/61.8
8	4.60/28.4	6.57/31.3	13.31/38.8	15.65/41.3
16	2.23/ 9.9	4.05/14.0	10.54/22.7	11.99/23.5
25	-1.34/-4.8	-.52/-1.4	10.52/18.5	14.82/23.8

I - 33

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 3:1 \quad L = 4 \quad K = 32 \quad 1 - \Phi(\tau-1) \approx .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	1.58/55.6	2.29/57.6	6.02/71.7	6.94/70.6
.3	3.15/57.7	4.29/61.2	8.72/70.4	10.16/72.6
1	2.77/27.3	3.87/31.0	7.71/39.4	8.93/41.3
2	3.23/22.6	4.36/25.2	8.49/32.6	9.75/34.3
4	3.19/15.9	4.32/17.9	8.00/22.7	8.86/23.2
8	3.44/12.2	4.69/13.9	9.31/19.4	10.87/21.0
16	2.71/ 6.8	3.90/ 8.3	8.42/12.7	9.78/13.8
25	1.89/ 3.8	3.09/ 5.3	8.57/10.5	10.65/12.2

I - 34

$\sigma^2:\mu = 3:1 \quad L = 4 \quad K = 32 \quad 1 - \Phi(\tau-1) \approx .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	4.91/48.4	6.45/51.7	11.47/58.6	12.94/59.9
2	5.19/36.3	6.78/39.1	11.98/46.0	13.54/47.6
4	5.75/28.6	7.45/30.9	12.81/36.4	14.35/37.6
8	6.27/22.2	8.07/24.0	14.19/29.5	16.17/31.2
16	4.76/12.0	6.29/13.4	11.81/17.8	12.95/18.2
25	2.11/ 4.3	3.37/ 5.8	10.43/12.8	13.36/15.3

I - 35

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 3:1$ $L = 4$ $K = 64$ $1 - \Phi(\tau-1) \approx .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	1.86/51.2	2.62/55.8	6.14/68.1	7.23/69.1
.3	3.69/54.8	4.67/56.4	9.04/66.4	10.58/69.2
1	3.06/24.5	3.89/26.1	7.78/35.3	8.99/37.3
2	3.51/19.9	4.62/22.2	8.62/29.0	9.93/30.9
4	3.45/13.8	4.53/15.5	8.11/20.0	8.89/20.4
8	3.78/10.7	4.97/12.1	9.44/16.9	11.00/18.5
16	3.04/ 6.1	4.14/ 7.2	8.41/10.9	9.66/11.7
25	2.38/ 3.8	3.47/ 4.9	8.98/ 9.4	11.05/10.9

I - 36

$\sigma^2:\mu = 3:1$ $L = 4$ $K = 64$ $1 - \Phi(\tau-1) \approx .15$

Service Levels

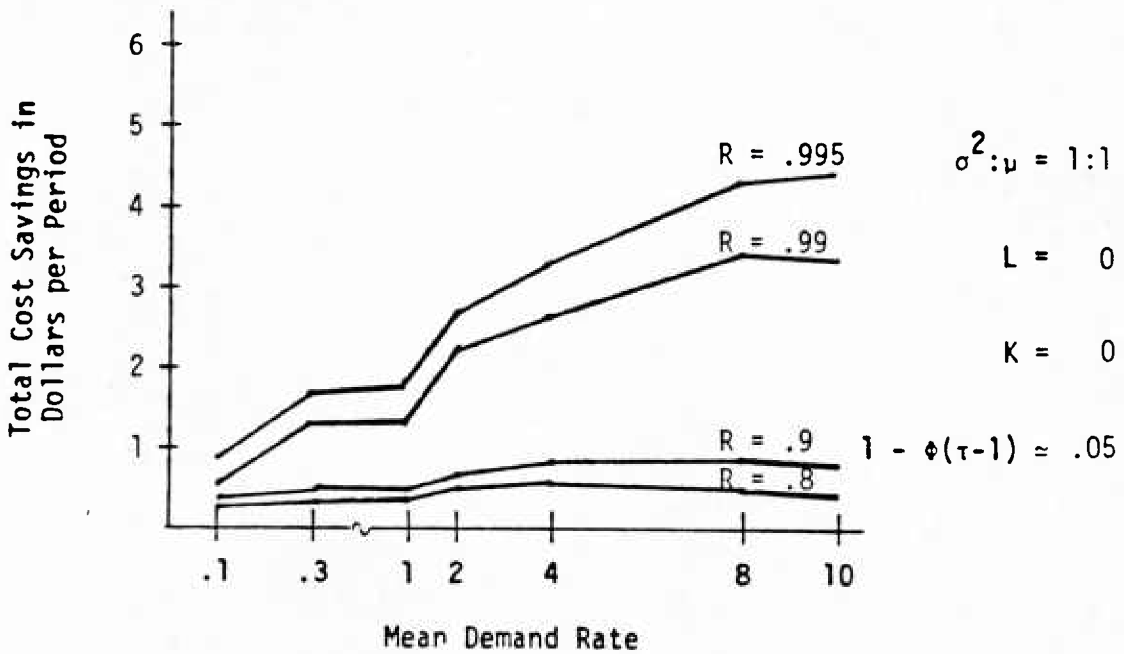
Mean Demand	R = .8	R = .9	R = .99	R = .995
1	5.65/45.1	7.20/48.2	12.19/55.3	13.63/56.5
2	5.87/33.2	7.43/35.7	12.64/42.5	14.19/44.1
4	6.55/26.2	8.20/28.1	13.56/33.5	14.99/34.4
8	7.37/20.9	9.14/22.3	15.18/27.2	17.11/28.7
16	6.29/12.7	7.70/13.4	12.76/16.5	13.80/16.8
25	3.87/ 6.3	4.82/ 6.8	12.05/12.6	15.03/14.8

I - 37

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels



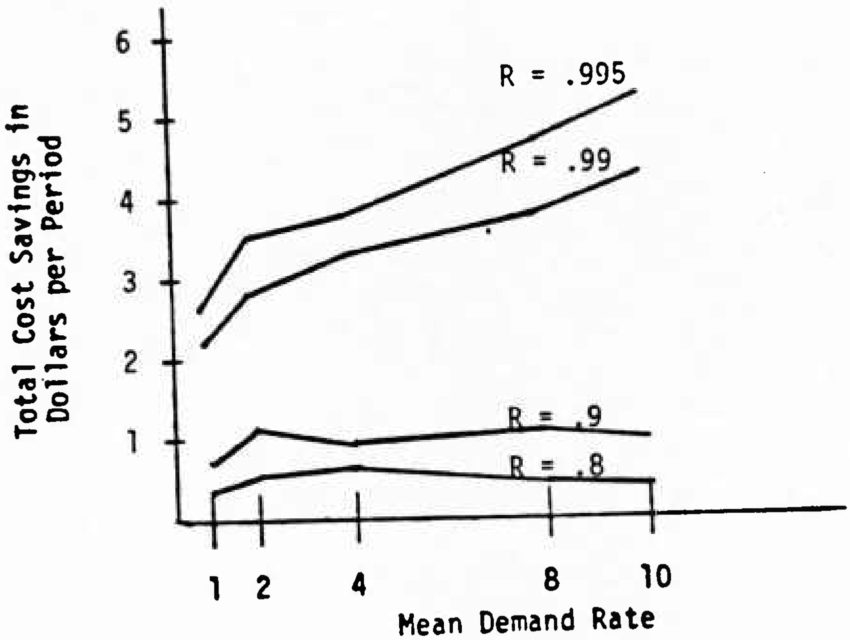
Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	.04/ 9.5	.09/ 9.5	0.47/34.0	0.95/51.0
.3	.13/14.0	.33/29.8	1.31/62.8	1.70/68.6
1	.13/ 8.6	.34/16.8	1.35/39.3	1.78/46.1
2	.35/16.9	.56/20.5	2.30/50.1	2.89/55.8
4	.42/14.1	.81/21.1	2.78/44.7	3.38/49.6
8	.39/ 9.4	.92/17.3	3.52/41.6	4.29/46.3
10	.37/ 8.1	.83/14.1	3.51/37.6	4.41/43.1

APPENDIX I

Reduction in Total Costs for Different Service Levels



$\sigma^2 : \mu = 1:1$

$L = 0$

$K = 0$

$1 - \phi(\tau - 1) \simeq .15$

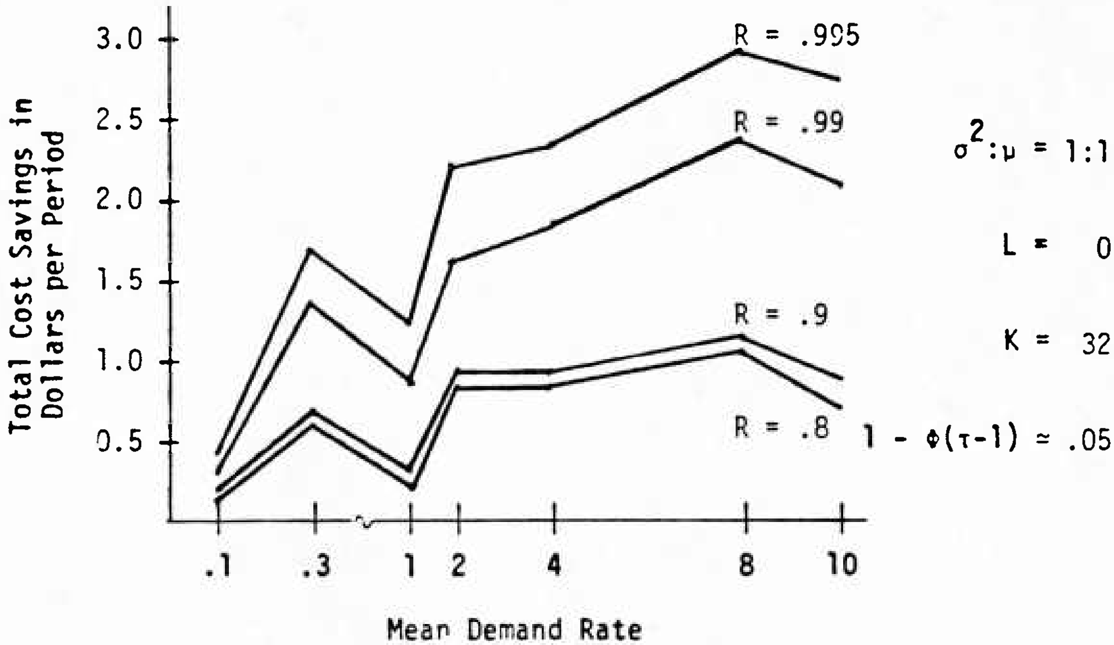
<u>Service Levels</u>

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	.33/22.0	0.77/37.9	2.17/63.1	2.60/67.1
2	.54/25.9	1.10/40.1	2.94/64.1	3.53/68.2
4	.60/20.0	0.98/25.6	3.32/53.4	3.88/56.8
8	.44/10.6	1.10/20.8	3.94/46.6	4.71/50.8
10	.33/ 7.1	1.06/18.1	4.26/45.6	5.16/50.4

APPENDIX I

Reduction in Total Costs for Different Service Levels



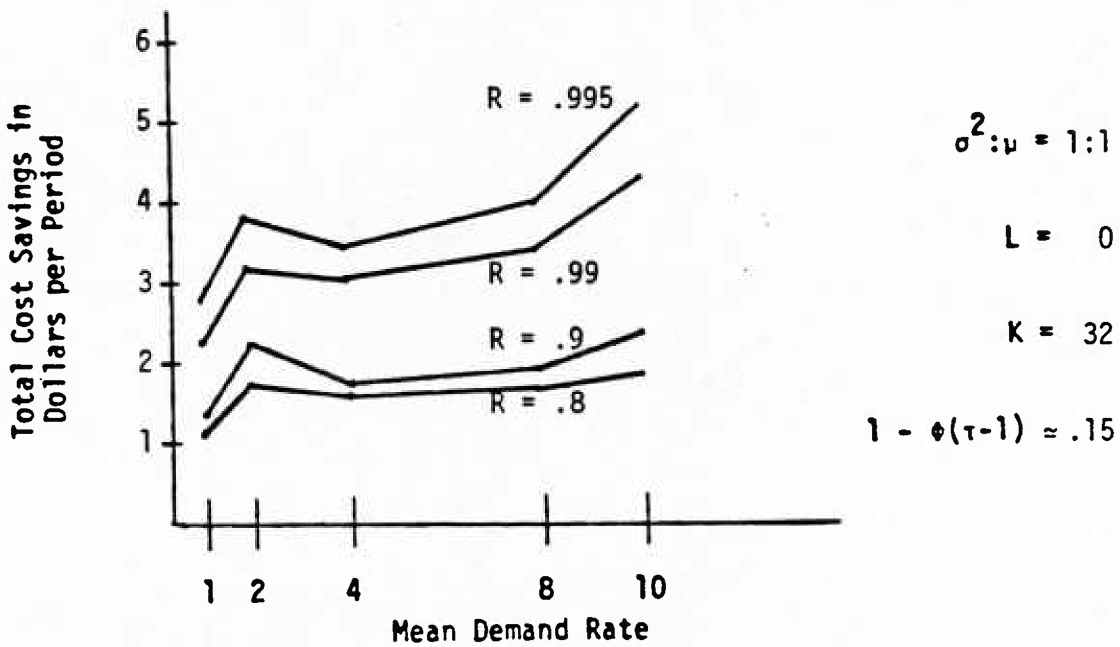
Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	0.11/ 5.3	0.13/ 5.8	0.27/ 8.6	0.43/12.0
.3	0.62/15.7	0.68/15.9	1.44/26.1	1.71/29.6
1	0.33/ 4.5	0.37/ 4.6	0.82/ 8.4	1.27/12.1
2	0.81/ 7.8	0.96/ 8.4	1.59/11.6	2.21/15.5
4	0.82/ 5.6	0.95/ 5.9	1.92/10.0	2.37/11.8
8	1.04/ 5.0	1.21/ 5.3	2.42/ 8.9	2.90/10.4
10	0.79/ 3.4	0.91/ 3.6	2.19/ 7.3	2.75/ 8.8

APPENDIX I

Reduction in Total Costs for Different Service Levels



I - 41

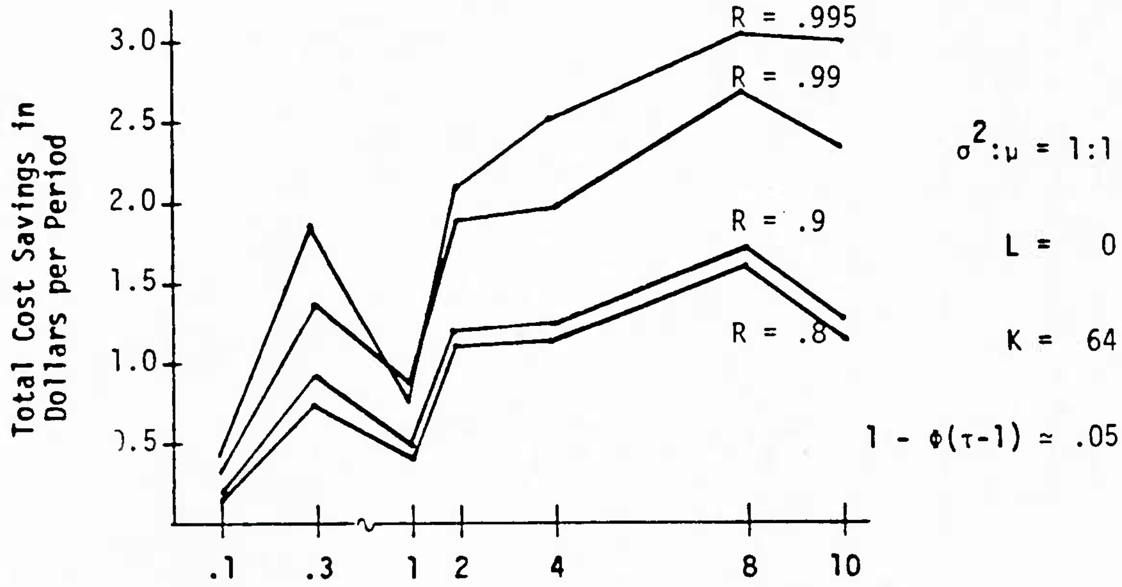
Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	1.10/15.0	1.22/15.1	2.19/22.3	2.88/27.4
2	1.88/18.1	2.14/18.7	3.22/23.6	3.84/26.9
4	1.62/11.0	1.81/11.2	3.01/15.6	3.48/17.4
8	1.67/ 8.0	1.89/ 8.3	3.37/12.5	3.96/14.1
10	1.93/ 8.3	2.24/ 8.8	4.33/14.4	5.09/16.3

APPENDIX I

Reduction in Total Costs for Different Service Levels



I - 42

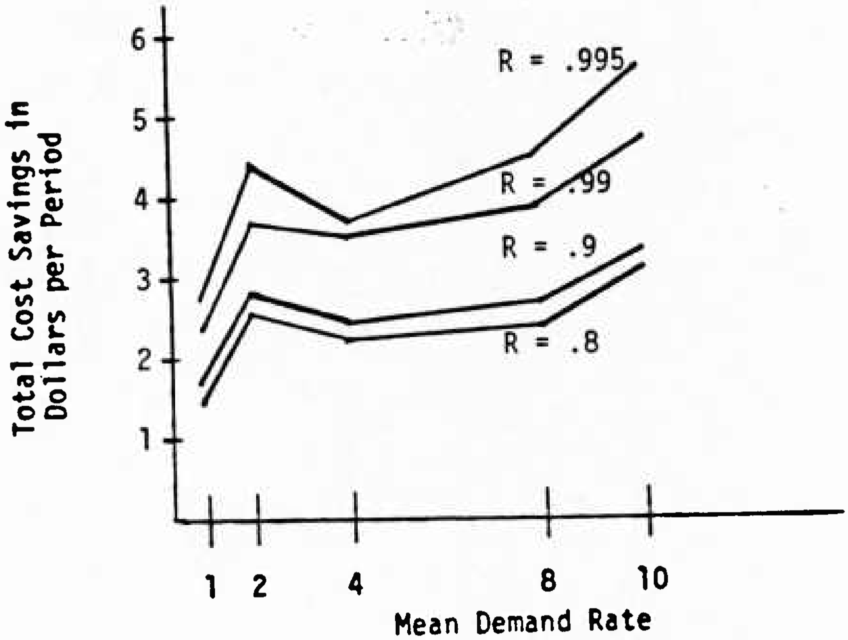
Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	0.16/ 5.1	0.18/ 5.4	0.28/ 6.8	0.40/ 9.3
.3	0.76/13.6	0.91/15.3	1.46/20.6	1.90/25.3
1	0.44/ 4.3	0.50/ 4.5	0.93/ 7.2	0.85/ 6.4
2	1.11/ 7.6	1.21/ 7.7	1.93/10.7	2.14/11.4
4	1.17/ 5.7	1.29/ 5.8	2.00/ 7.9	2.51/ 9.6
8	1.53/ 5.3	1.65/ 5.3	2.67/ 7.5	3.06/ 8.3
10	1.23/ 3.8	1.34/ 3.8	2.42/ 6.0	3.03/ 7.3

APPENDIX I

Reduction in Total Costs for Different Service Levels



$\sigma^2 : \mu = 1:1$

$L = 0$

$K = 64$

$1 - \Phi(\tau - 1) \simeq .15$

I - 43

Service Levels

Total Cost Savings/Percent Reduction in Total Costs

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	1.50/14.6	1.67/15.2	2.43/18.8	2.81/21.1
2	2.61/18.0	2.82/18.1	3.81/21.0	4.43/23.6
4	2.32/11.3	2.47/11.1	3.56/14.0	3.78/14.4
8	2.54/ 8.8	2.69/ 8.6	3.98/11.1	4.54/12.3
10	3.09/ 9.5	3.26/ 9.3	4.90/12.2	5.67/13.7

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 1:1$ $L = 1$ $K = 0$ $1 - \Phi(\tau-1) \simeq .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	.08/10.0	.09/ 8.7	0.28/14.4	0.20/ 9.8
.3	.34/30.0	.62/37.3	1.22/44.1	1.60/50.8
1	.25/12.1	.29/10.5	1.05/22.8	1.25/24.2
2	.55/18.5	.79/20.5	1.82/29.2	2.20/32.2
4	.47/11.3	.77/14.5	2.28/27.0	2.81/30.3
8	.35/ 6.0	.78/10.6	2.56/22.1	3.27/25.8
10	.26/ 4.0	.73/ 9.0	2.75/21.4	3.55/25.1

I - 44

$\sigma^2:\mu = 1:1$ $L = 1$ $K = 0$ $1 - \Phi(\tau-1) \simeq .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	.55/26.1	.88/32.1	2.06/44.8	2.64/51.0
2	.87/29.2	1.23/31.9	1.82/29.2	2.20/32.2
4	.59/14.3	1.03/19.4	2.64/31.2	3.35/36.1
8	.23/ 4.0	.76/10.3	3.06/26.4	3.62/28.5
10	-.18/-2.9	.49/ 6.0	3.30/25.6	3.94/27.9

I - 45

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 1:1$ $\quad L = 1$ $\quad K = 32$ $\quad 1 - \Phi(\tau-1) \simeq .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	.13/ 5.7	.17/ 6.7	0.45/12.9	0.60/15.1
.3	.63/15.4	.72/15.4	1.09/18.4	1.64/25.3
1	.36/ 4.7	.45/ 5.3	0.94/ 8.7	1.01/ 9.0
2	.86/ 8.0	1.12/ 9.3	1.82/12.2	1.97/12.6
4	.90/ 5.9	1.07/ 6.3	2.05/ 9.8	2.56/11.7
8	.99/ 4.6	1.18/ 4.9	2.41/ 8.2	2.77/ 9.0
10	.73/ 3.0	0.92/ 3.4	2.32/ 7.1	3.09/ 9.0

I - 46

$\sigma^2:\mu = 1:1$ $\quad L = 1$ $\quad K = 32$ $\quad 1 - \Phi(\tau-1) \simeq .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	1.20/15.7	1.40/16.4	2.34/21.9	2.36/21.1
2	2.01/18.6	2.31/19.2	3.47/23.3	3.83/24.5
4	1.67/10.9	1.96/11.5	3.02/14.4	3.41/15.6
8	1.57/ 7.3	1.79/ 7.5	3.35/11.4	3.88/12.7
10	1.67/ 6.9	1.85/ 6.9	4.02/12.3	4.96/14.5

I - 47

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 1:1$ $L = 1$ $K = 64$ $1 - \Phi(\tau-1) \simeq .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	0.17/ 5.3	0.18/ 5.3	0.41/ 9.3	0.68/13.9
.3	0.85/15.0	0.99/15.9	1.43/18.8	1.59/19.8
1	0.48/ 4.5	0.58/ 5.0	0.87/ 6.3	1.18/ 8.2
2	1.14/ 7.7	1.29/ 8.0	1.98/10.3	2.32/11.6
4	1.21/ 5.8	1.37/ 6.0	2.44/ 9.0	2.88/10.3
8	1.47/ 5.0	1.65/ 5.1	2.69/ 7.0	3.06/ 7.7
10	1.15/ 3.5	1.30/ 3.6	2.65/ 6.2	3.23/ 7.6

I - 48

$\sigma^2:\mu = 1:1$ $L = 1$ $K = 64$ $1 - \Phi(\tau-1) \simeq .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	1.56/14.9	1.82/15.9	2.67/19.5	2.90/20.3
2	2.69/18.2	3.01/18.6	4.08/21.1	4.60/22.9
4	2.34/11.1	2.59/11.3	3.68/13.6	3.98/14.2
8	2.42/ 8.2	2.59/ 8.0	3.95/10.4	4.47/11.3
10	2.82/ 8.5	2.92/ 8.1	4.74/11.1	5.58/12.7

I - 49

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 1:1$ L = 4 K = 0 $1 - \Phi(\tau-1) \approx .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	.11/11.0	0.27/17.1	0.45/16.5	0.20/ 7.1
.3	.59/30.8	0.65/27.0	1.17/28.9	1.74/37.6
1	.37/11.2	0.53/12.5	1.02/14.9	1.35/17.7
2	.77/16.7	1.05/17.9	2.02/21.6	2.11/20.6
4	.65/10.1	1.02/12.4	2.49/19.3	3.20/22.7
8	.13/ 1.5	0.56/ 4.9	2.07/11.6	2.40/12.3
10	-.02/-0.2	0.43/ 3.4	2.60/13.1	3.75/17.3

I - 50

$\sigma^2:\mu = 1:1$ L = 4 K = 0 $1 - \Phi(\tau-1) \approx .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	0.92/ 27.9	1.23/29.2	2.26/49.2	2.75/36.1
2	1.26/ 27.2	1.69/28.8	3.14/33.6	3.53/34.4
4	0.80/ 12.5	1.24/15.1	2.55/19.9	2.79/19.8
*8	-0.26/ -2.8	0.19/ 1.7	2.17/12.2	2.76/14.2
10	-1.24/-12.1	-0.76/-6.0	1.94/ 9.8	3.20/14.8

$*1 - \Phi(\tau-1) = .1841$

I - 51

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 1:1$ $L = 4$ $K = 32$ $1 - \Phi(\tau-1) \approx .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	0.18/ 6.9	0.19/ 6.3	0.39/ 9.3	0.65/13.8
.3	0.73/16.2	0.91/17.5	1.64/23.3	1.96/25.7
1	0.46/ 5.5	0.62/ 6.5	1.09/ 8.7	1.15/ 8.6
2	1.06/ 9.1	1.32/17.7	2.06/11.8	2.10/11.3
4	1.04/ 6.3	1.32/ 7.0	2.42/ 9.9	2.90/11.2
8	0.89/ 3.8	1.12/ 4.2	2.20/ 6.4	2.46/ 6.8
10	0.57/ 2.2	0.80/ 2.7	2.21/ 5.8	2.75/ 6.8

I - 52

$\sigma^2:\mu = 1:1$ $L = 4$ $K = 32$ $1 - \Phi(\tau-1) \approx .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	1.40/16.9	0.62/ 6.5	2.69/21.5	3.08/23.2
2	2.28/19.4	2.74/20.4	4.00/22.9	4.40/23.8
4	1.82/11.0	2.17/11.5	3.16/12.9	3.32/12.8
*8	1.29/ 5.5	1.52/ 5.7	2.97/ 8.6	3.48/ 9.6
10	1.02/ 3.9	1.12/ 3.8	2.67/ 7.0	3.38/ 8.4

I - 53

*$1 - \Phi(\tau-1) = .1841$

Total Cost Savings/Percent Reduction in Total Costs

APPENDIX I

Reduction in Total Costs for Different Service Levels

$\sigma^2:\mu = 1:1$ $L = 4$ $K = 64$ $1 - \Phi(\tau-1) \approx .05$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
.1	0.21/ 6.1	0.21/ 5.3	0.34/ 6.8	0.58/10.5
.3	0.96/16.0	1.22/17.9	1.77/20.4	2.06/22.6
1	0.53/ 4.8	0.68/ 5.5	1.34/ 7.3	1.23/ 7.6
2	1.29/ 8.3	1.58/ 9.1	2.29/10.5	2.32/10.2
4	1.31/ 6.0	1.57/ 6.4	2.63/ 8.6	2.96/ 9.3
8	1.34/ 4.3	1.54/ 4.4	2.56/ 6.0	2.74/ 6.1
10	0.97/ 2.8	1.14/ 2.9	2.44/ 5.1	2.97/ 5.9

I - 54

$\sigma^2:\mu = 1:1$ $L = 4$ $K = 64$ $1 - \Phi(\tau-1) \approx .15$

Service Levels

Mean Demand	R = .8	R = .9	R = .99	R = .995
1	1.75/15.9	2.05/16.6	3.16/20.3	3.35/20.6
2	2.91/18.7	3.37/19.4	4.73/21.8	4.99/21.9
4	2.42/11.0	2.76/11.3	3.75/12.3	3.79/11.9
* 8	2.12/ 6.8	2.32/ 6.7	3.63/ 8.5	4.16/ 9.3
10	2.15/ 6.2	2.19/ 5.7	3.59/ 8.1	4.28/ 8.6

I - 55

*$1 - \Phi(\tau-1) = .1841$

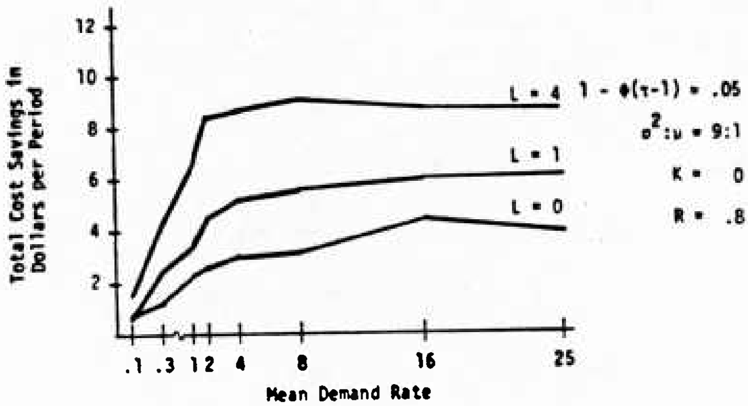
Total Cost Savings/Percent Reduction in Total Costs

APPENDIX II

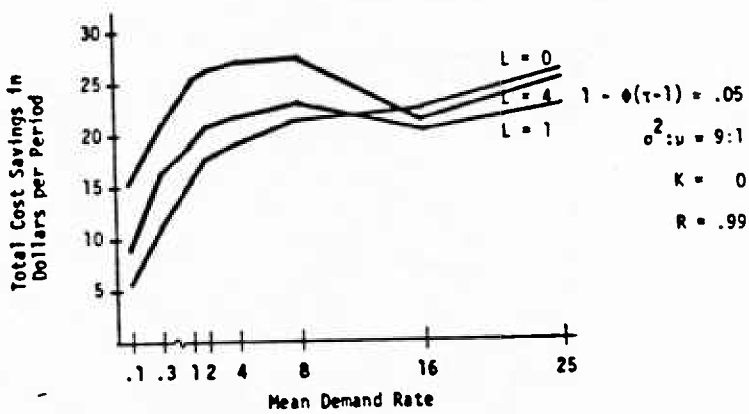
REDUCTION IN TOTAL INVENTORY COSTS FOR DIFFERENT LEADTIME

APPENDIX II

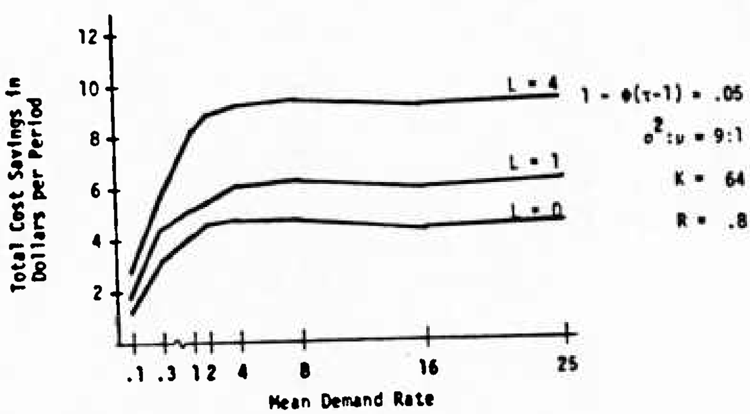
Reduction in Total Inventory Costs for Different Leadtime



II - 1



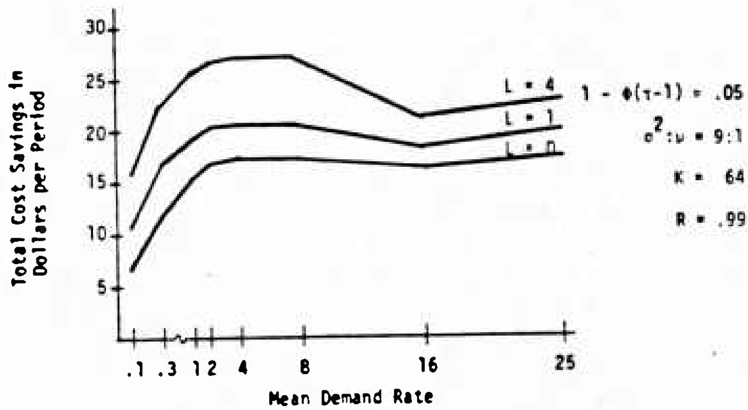
II - 2



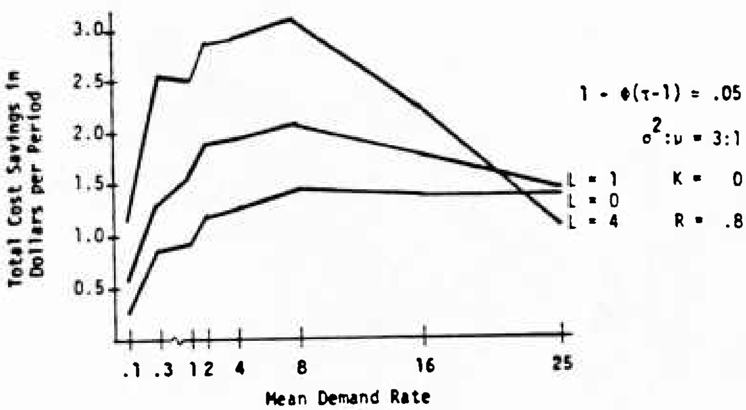
II - 3

APPENDIX II

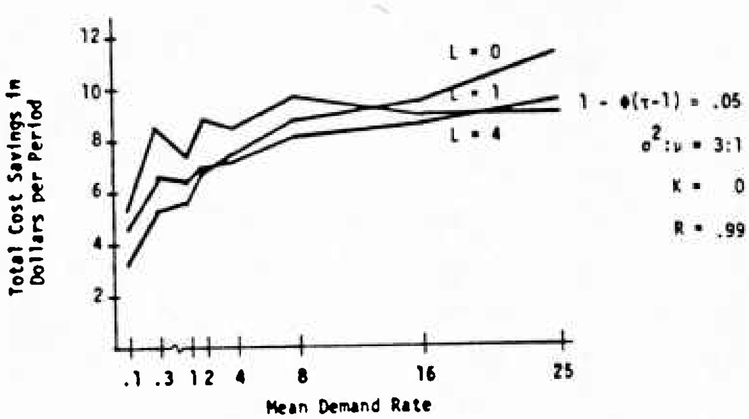
Reduction in Total Inventory Costs for Different Leadtime



II - 4



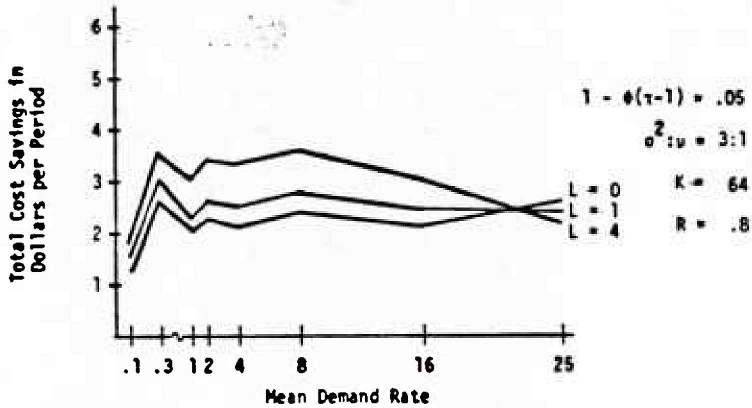
II - 5



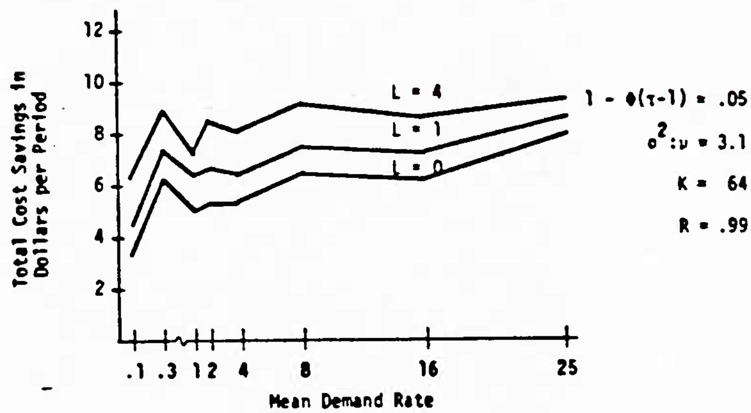
II - 6

APPENDIX II

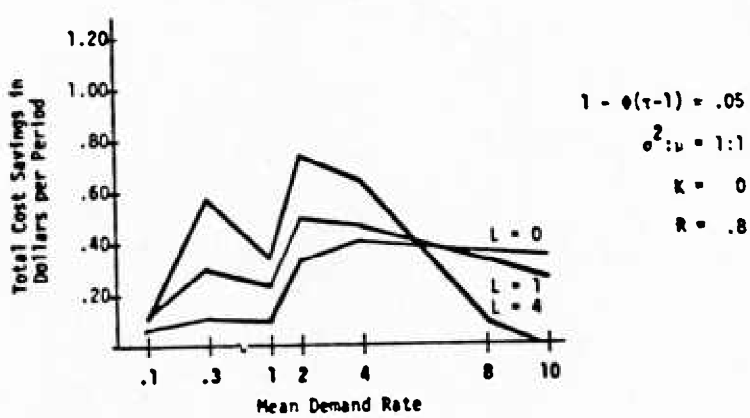
Reduction in Total Inventory Costs for Different Leadtime



II - 7



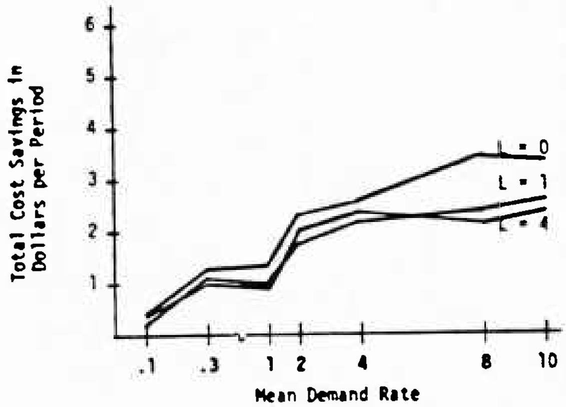
II - 8



II - 9

APPENDIX II

Reduction in Total Inventory Costs for Different Leadtime



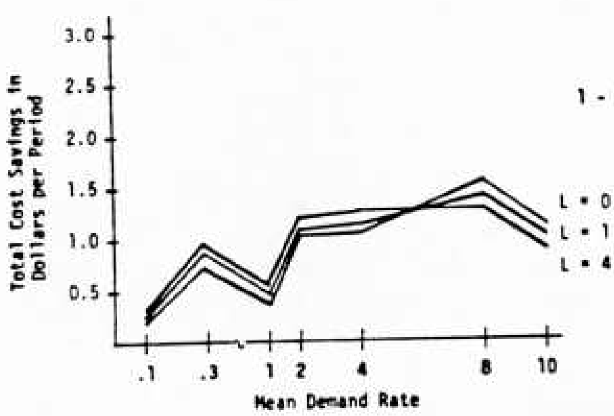
1 - Φ(τ-1) = .05

$\sigma^2:\mu$ = 1:1

K = 0

R = .99

II - 10



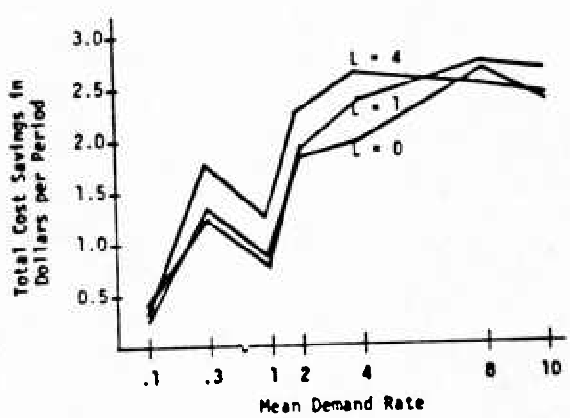
1 - Φ(τ-1) = .05

$\sigma^2:\mu$ = 1:1

K = 64

R = .8

II - 11



1 - Φ(τ-1) = .05

$\sigma^2:\mu$ = 1:1

K = 64

R = .99

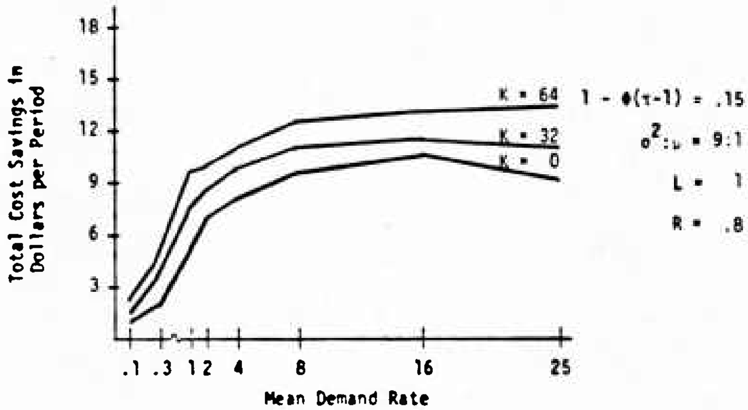
II -12

APPENDIX III

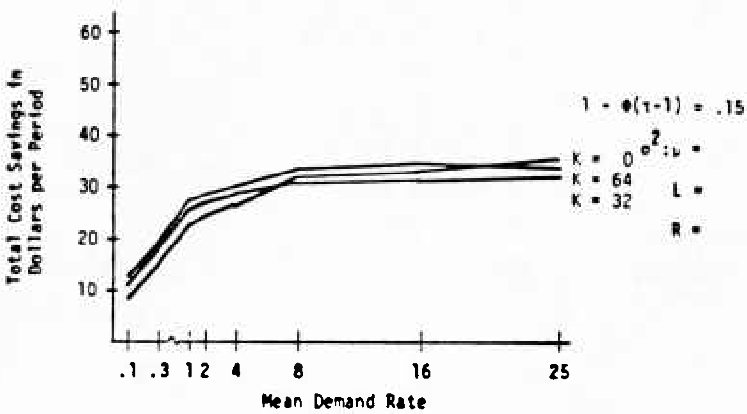
REDUCTION IN TOTAL INVENTORY COSTS FOR DIFFERENT SET UP COSTS

APPENDIX III

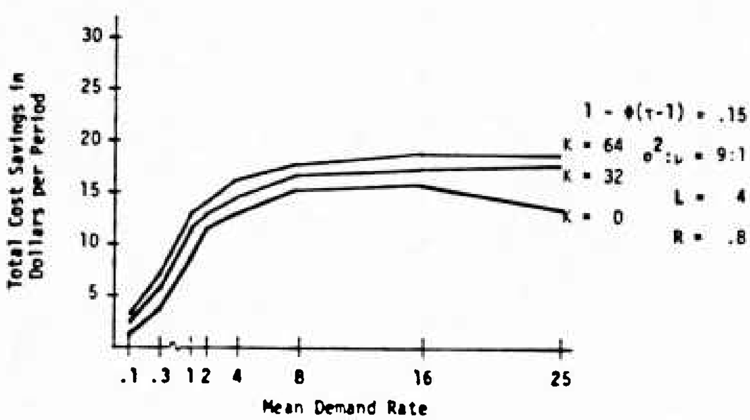
Reduction in Total Inventory Costs for Different Set Up Costs



III - 1



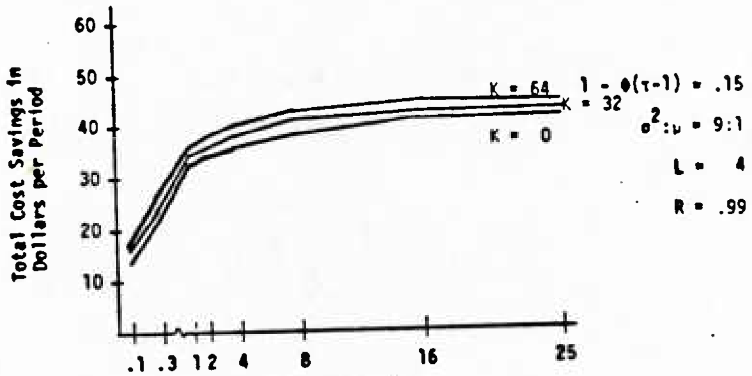
III - 2



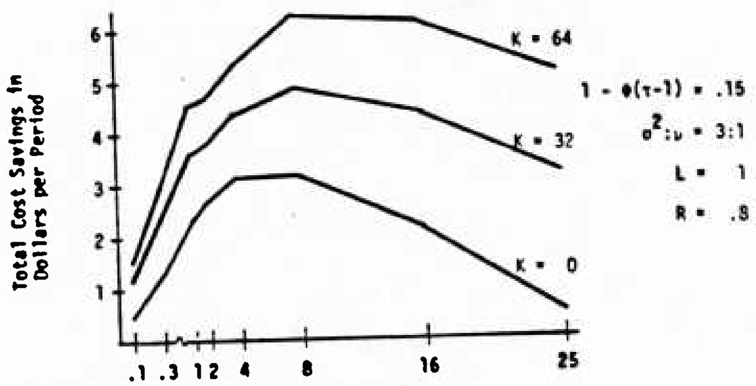
III - 3

APPENDIX III

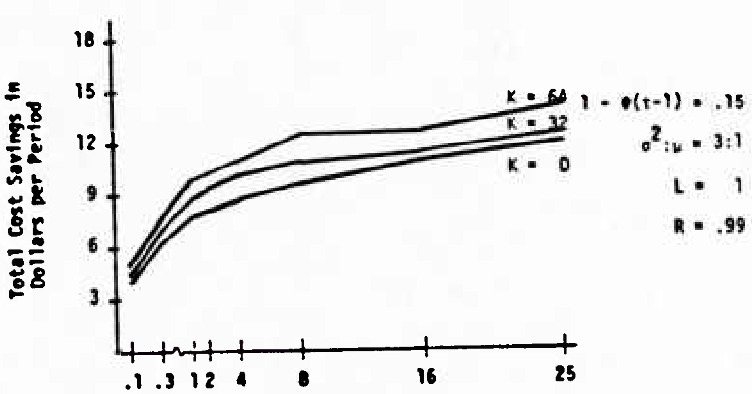
Reduction in Total Inventory Costs for Different Set Up Costs



III - 4



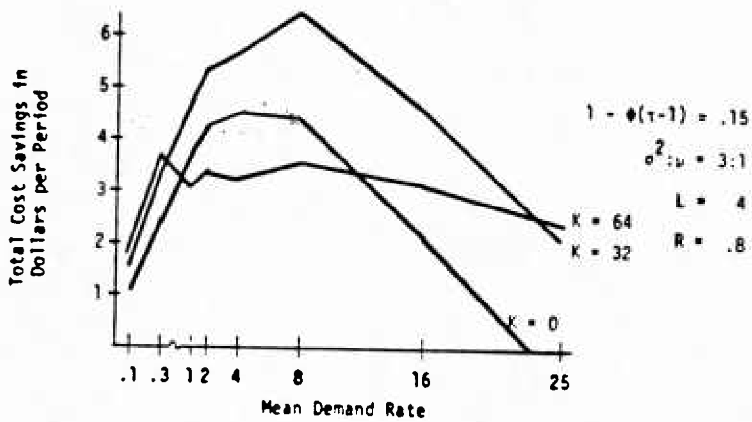
III - 5



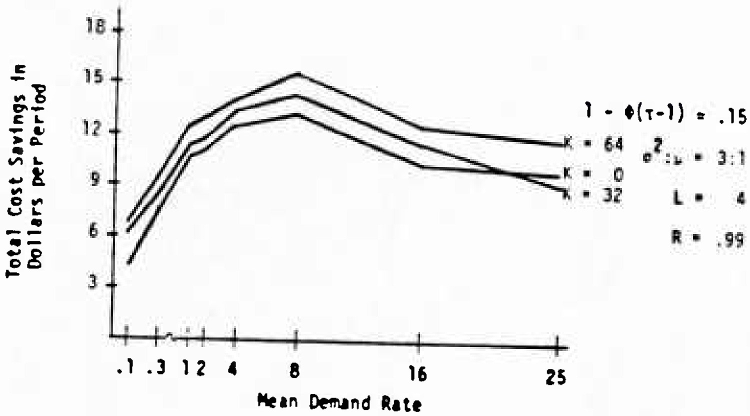
III - 6

APPENDIX III

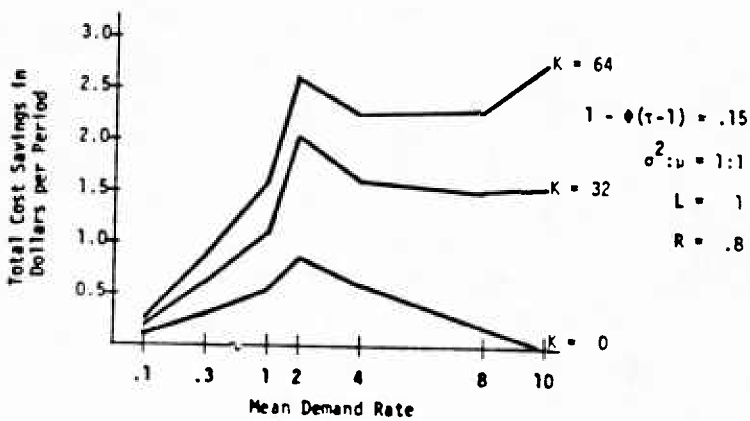
Reduction in Total Inventory Costs for Different Set Up Costs



III - 7



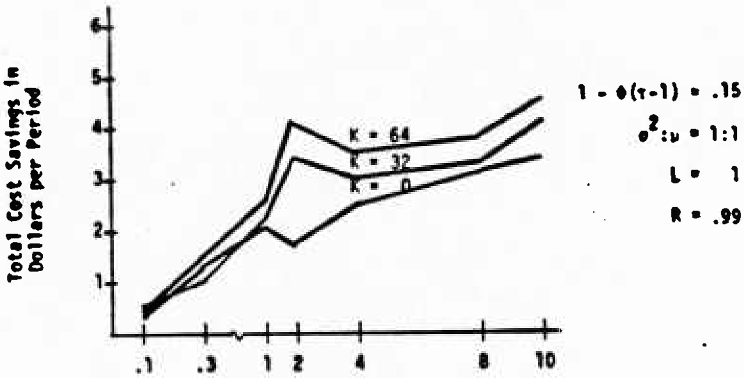
III - 8



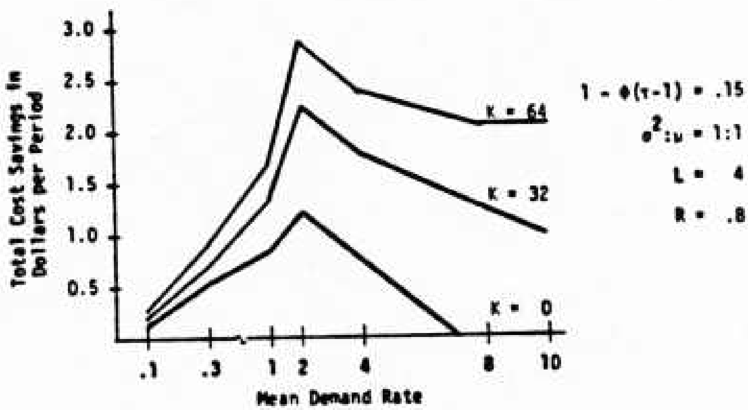
III - 9

APPENDIX III

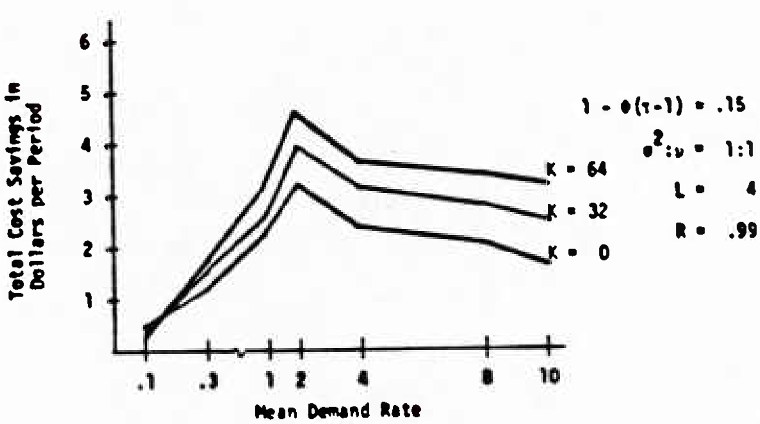
Reduction in Total Inventory Costs for Different Set Up Costs



III - 10



III - 11



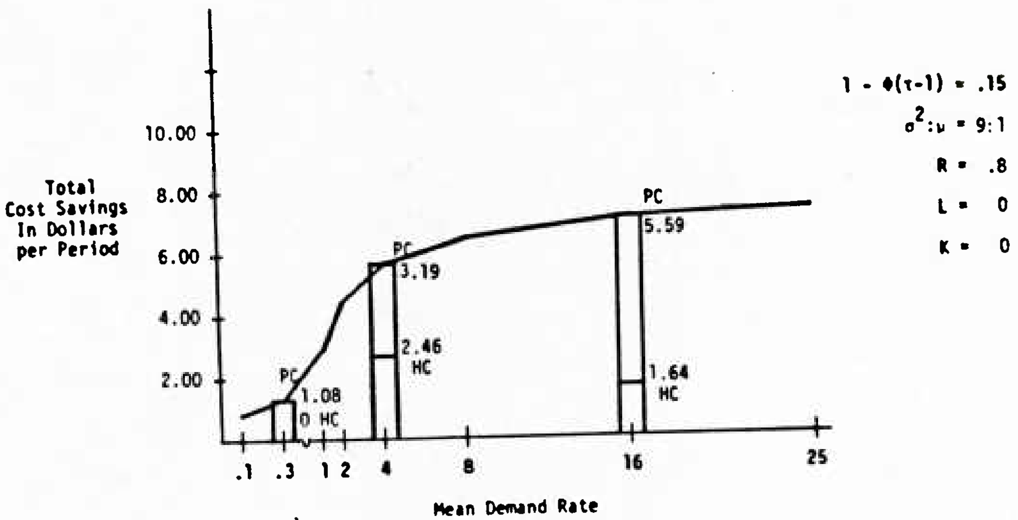
III - 12

APPENDIX IV

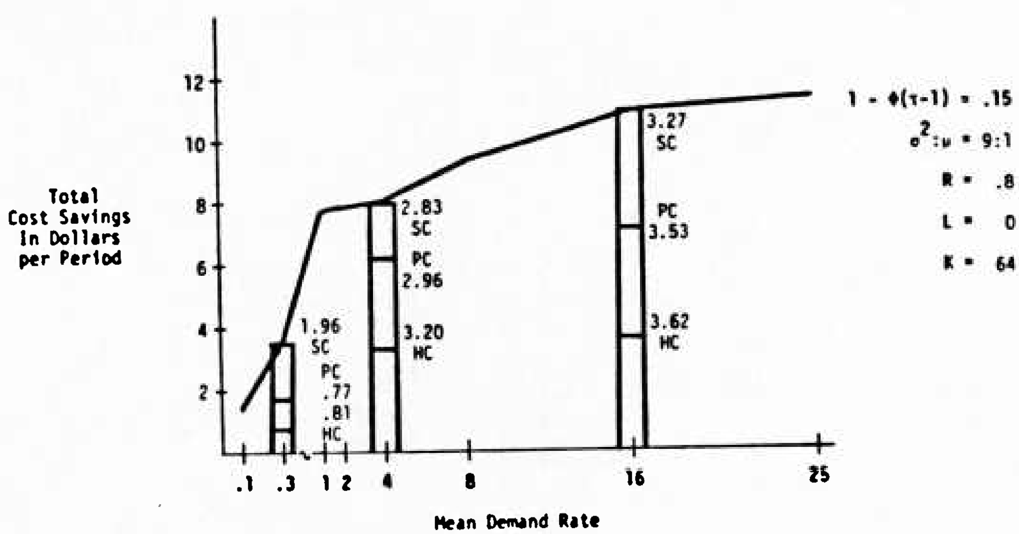
CATEGORIZATION OF TOTAL COST SAVINGS

APPENDIX IV

Categorization of Total Cost Savings



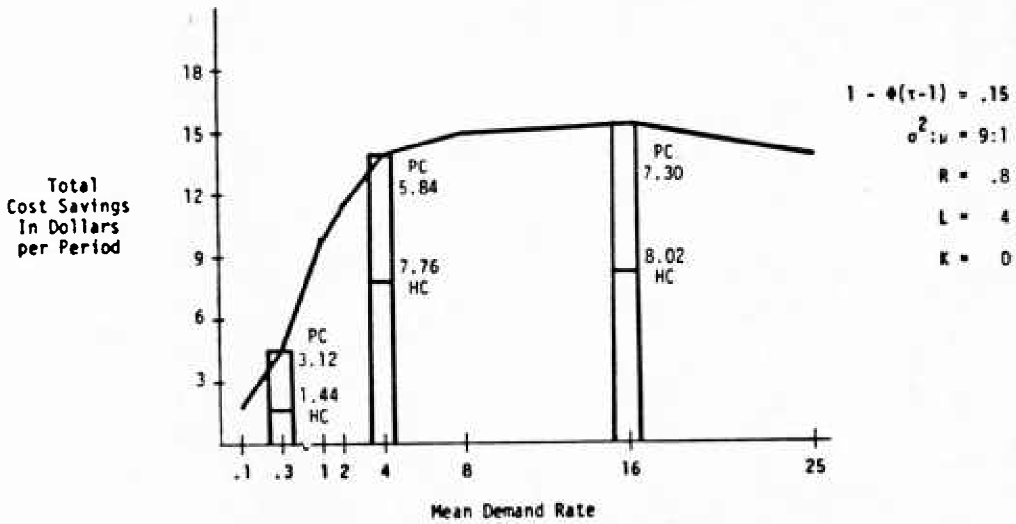
IV - 1



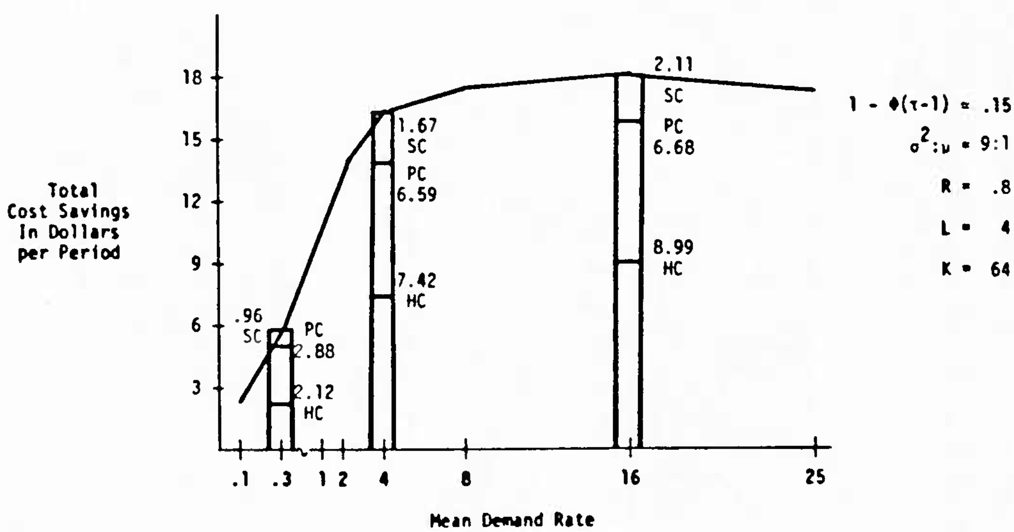
IV - 2

APPENDIX IV

Categorization of Total Cost Savings



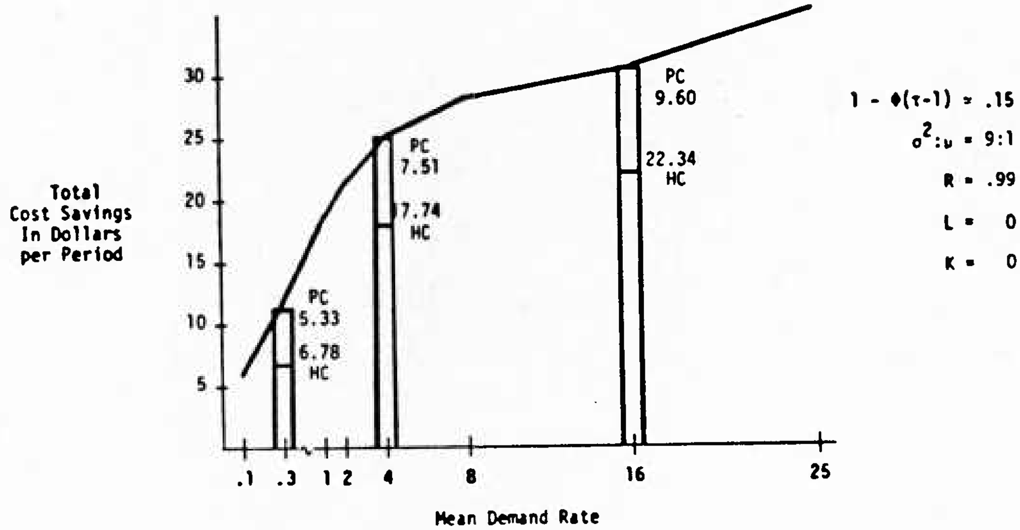
IV - 3



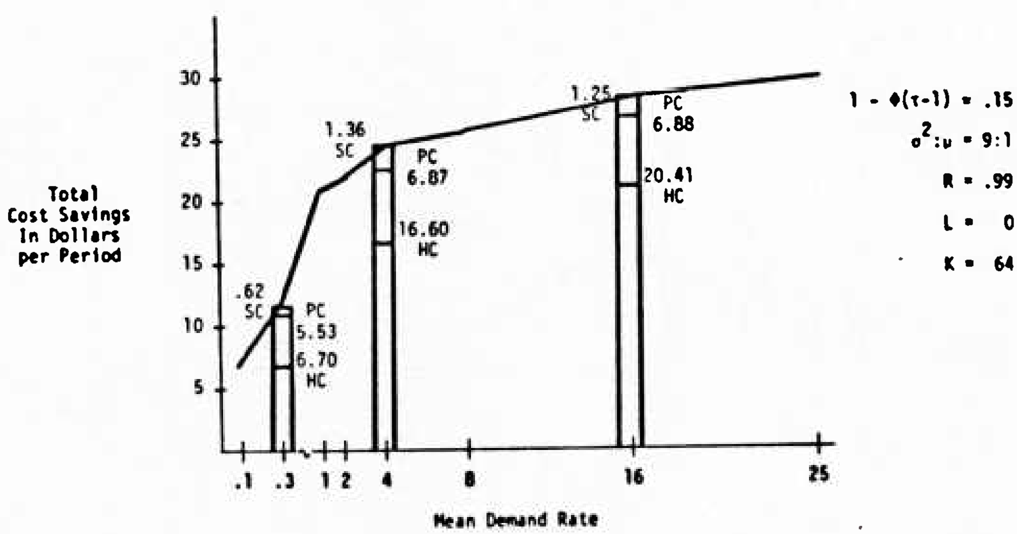
IV - 4

APPENDIX IV

Categorization of Total Cost Savings



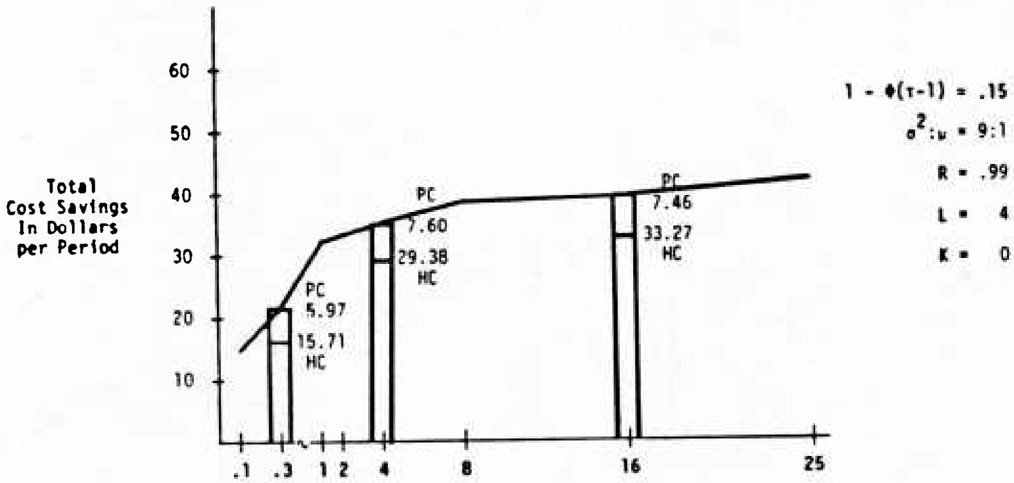
IV - 5



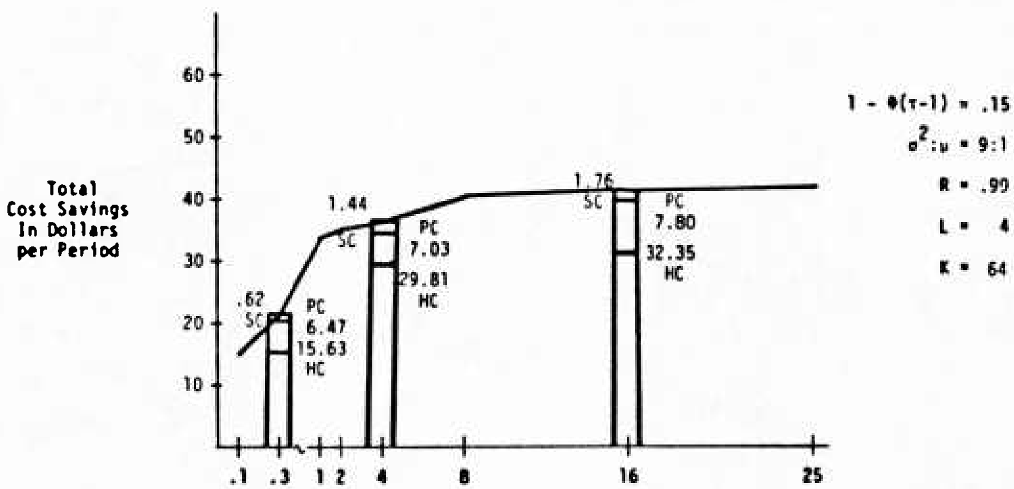
IV - 6

APPENDIX IV

Categorization of Total Cost Savings



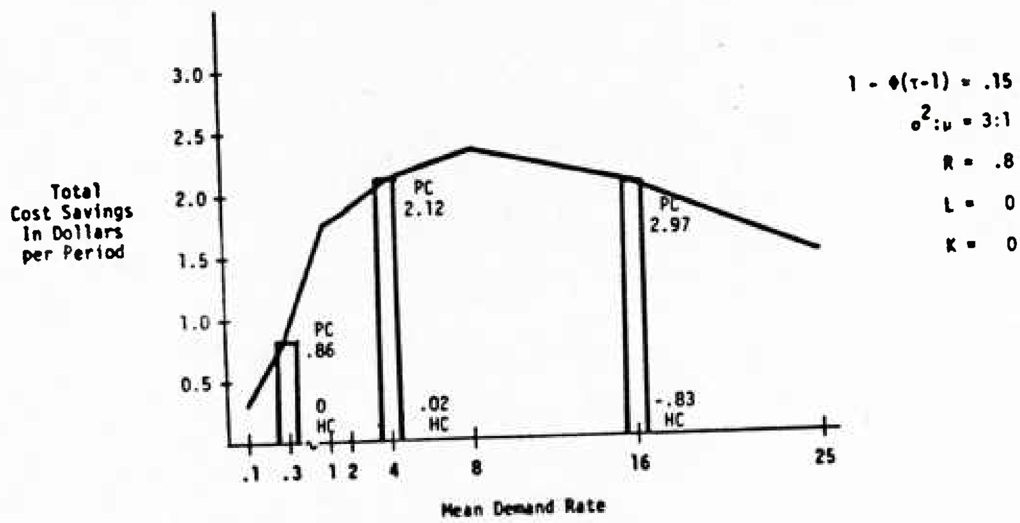
IV - 7



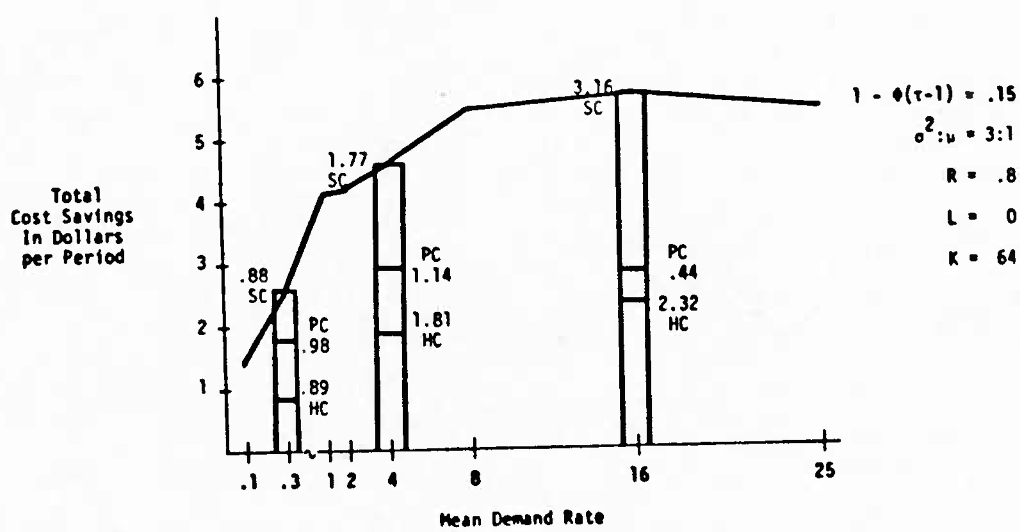
IV - 8

APPENDIX IV

Categorization of Total Cost Savings



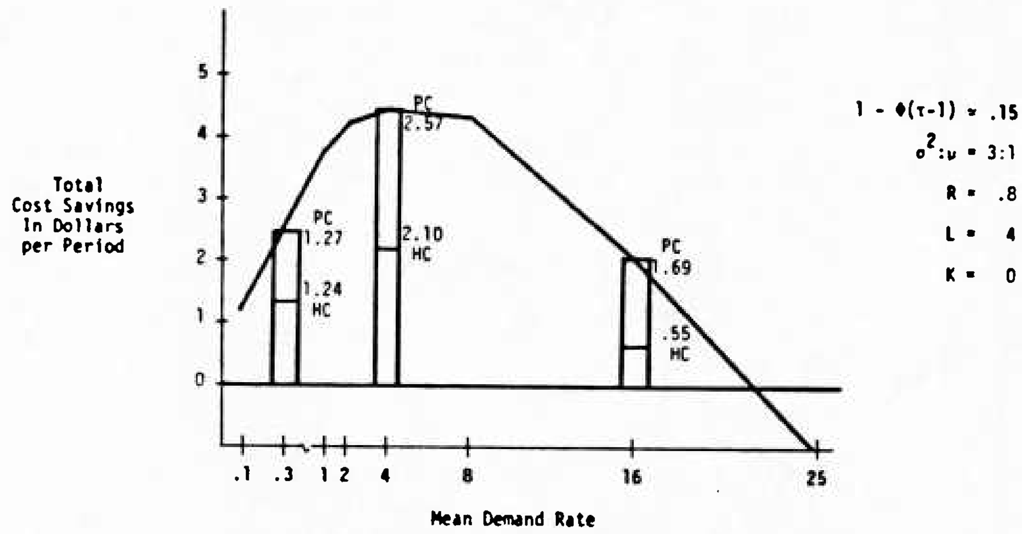
IV - 9



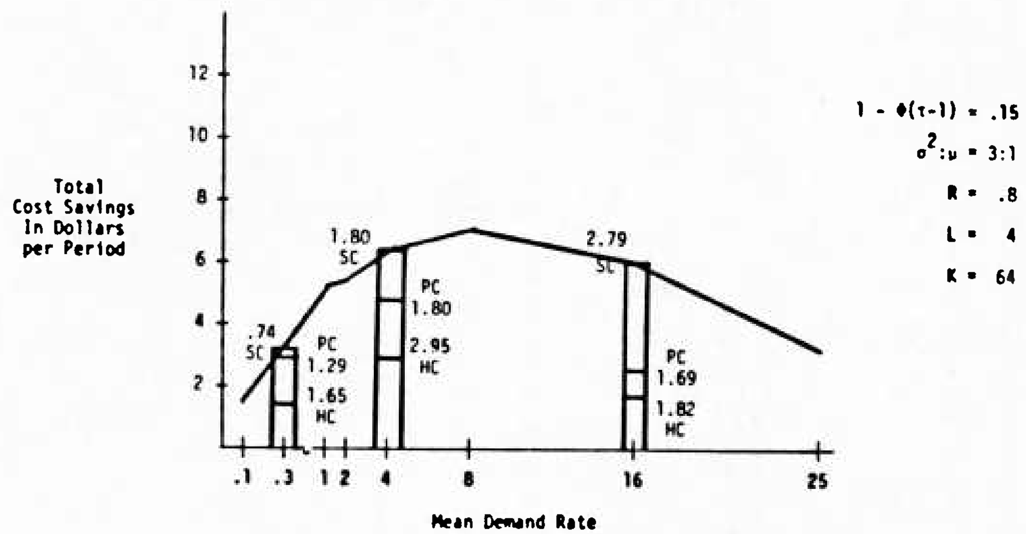
IV - 10

APPENDIX IV

Categorization of Total Cost Savings



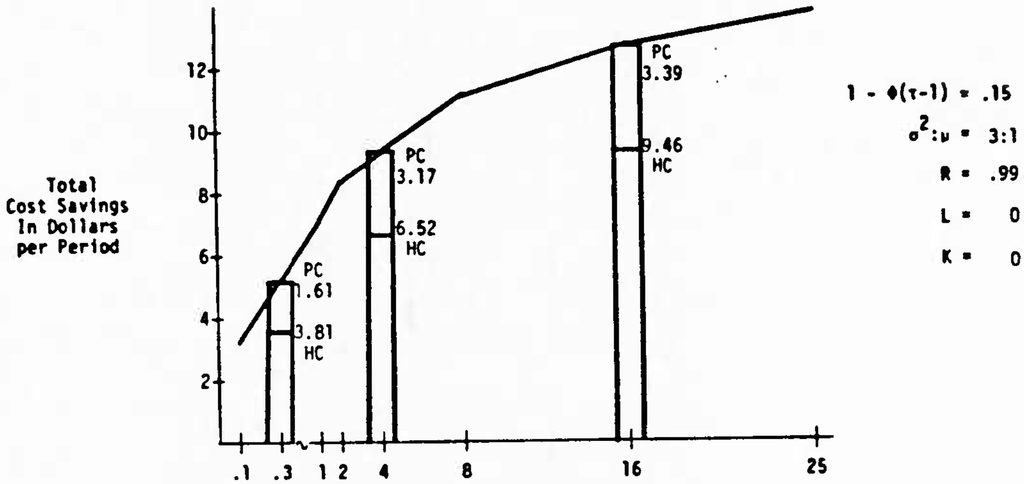
IV - 11



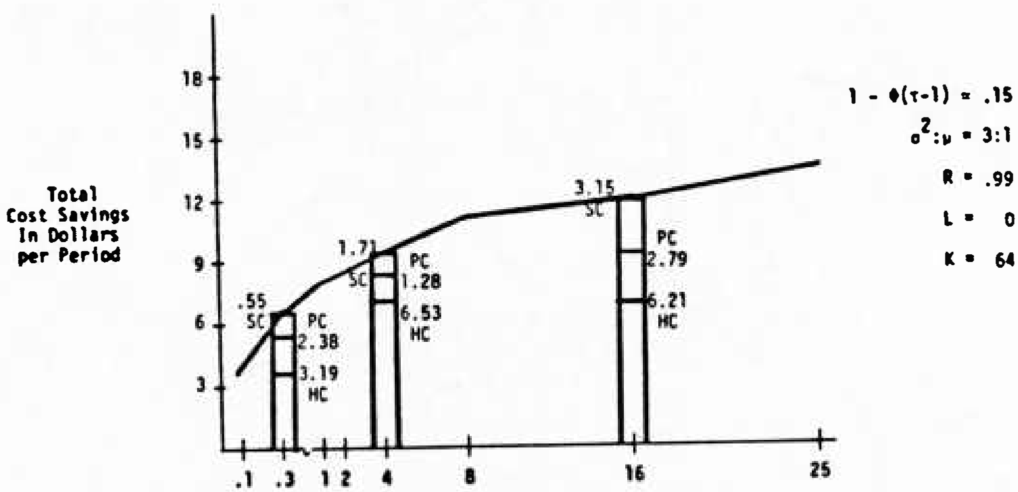
IV - 12

APPENDIX IV

Categorization of Total Cost Savings



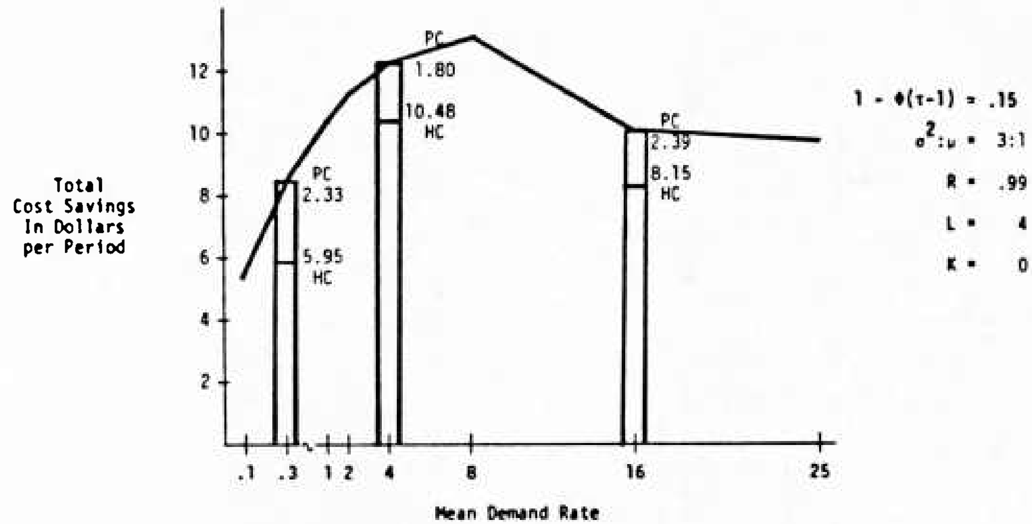
IV - 13



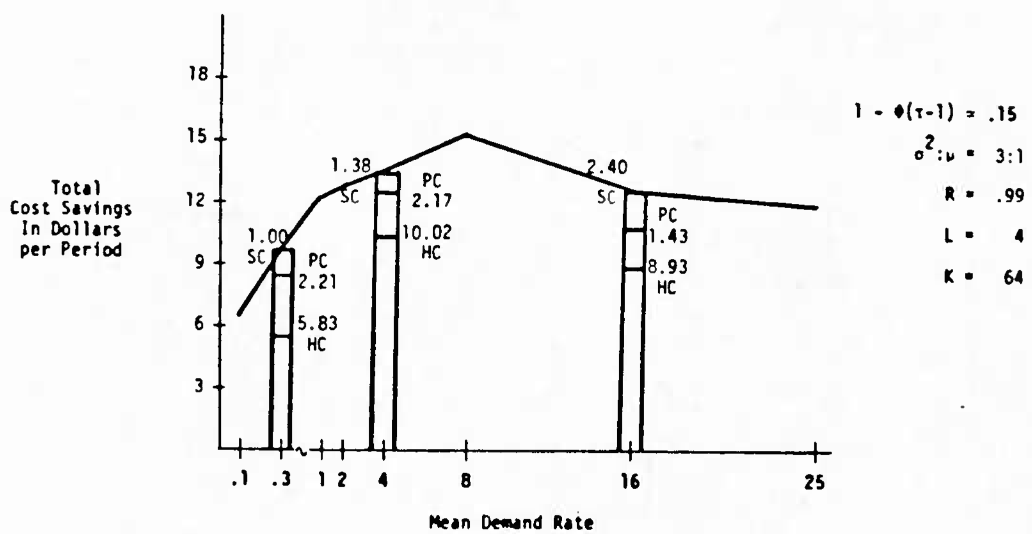
IV - 14

APPENDIX IV

Categorization of Total Cost Savings



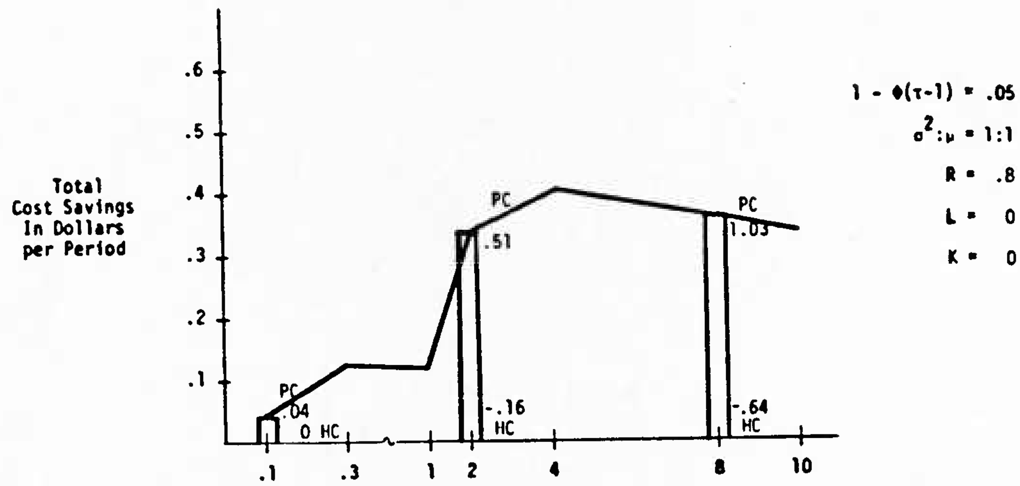
IV - 15



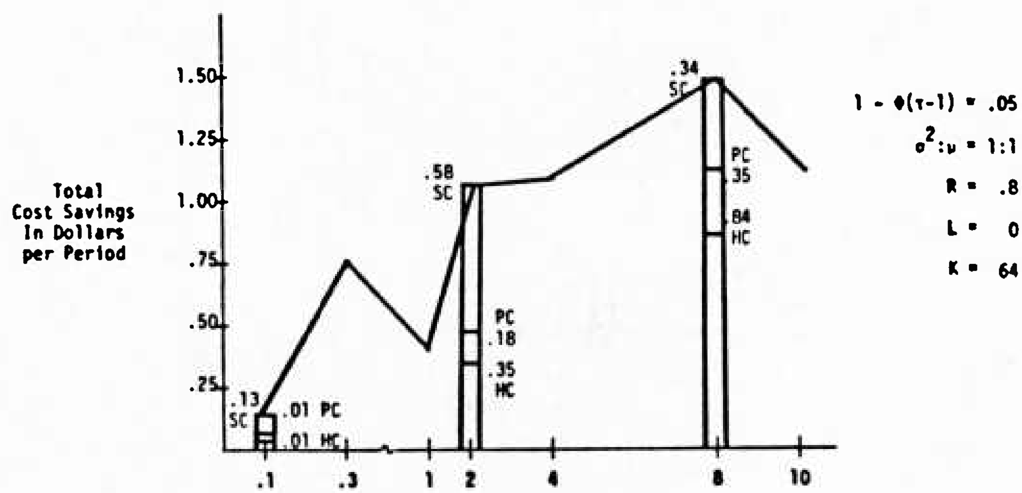
IV - 16

APPENDIX IV

Categorization of Total Cost Savings



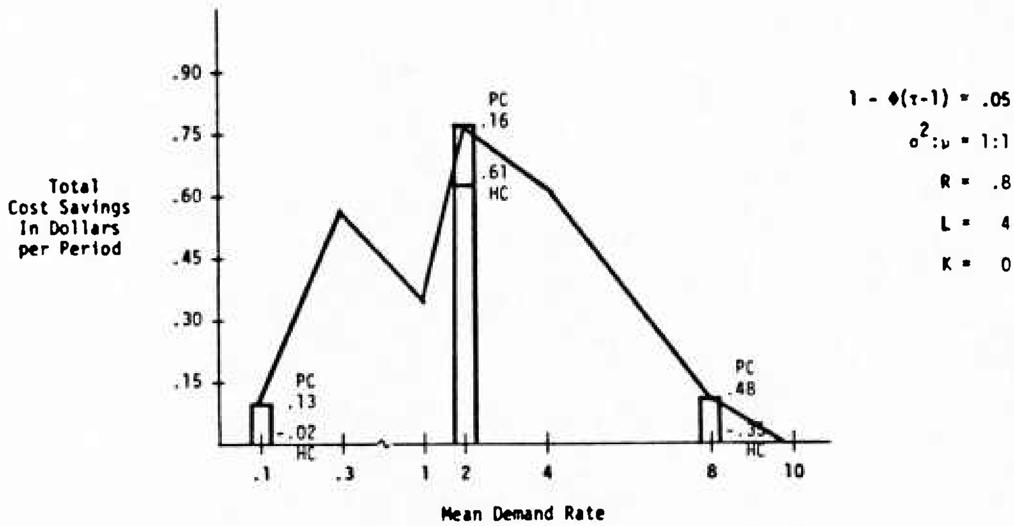
IV - 17



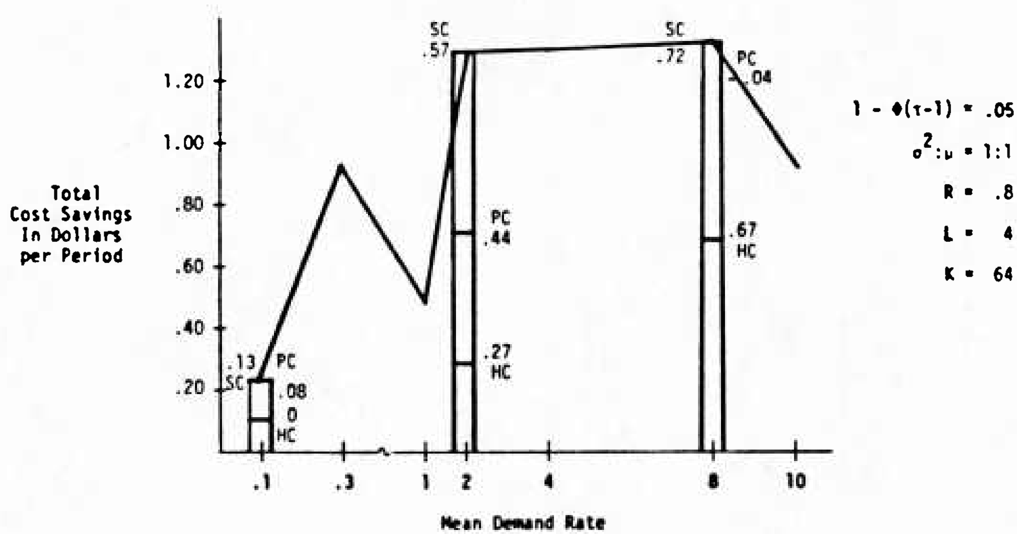
IV - 18

APPENDIX IV

Categorization of Total Cost Savings



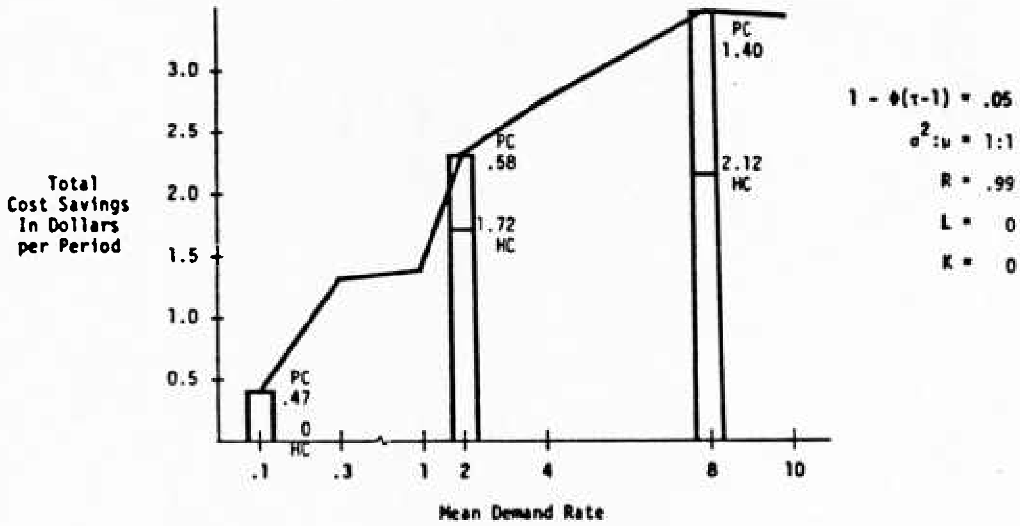
IV - 19



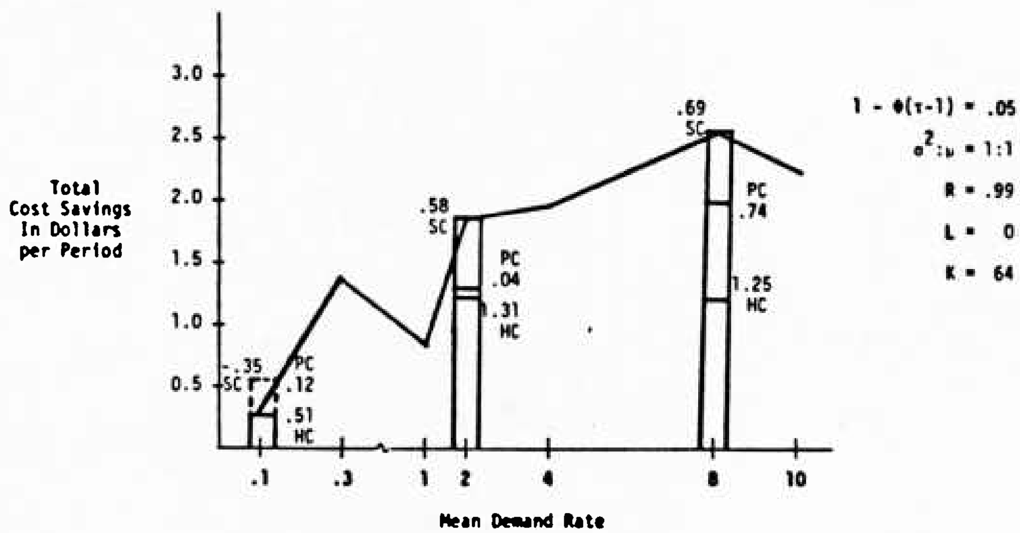
IV - 20

APPENDIX IV

Categorization of Total Cost Savings



IV - 21

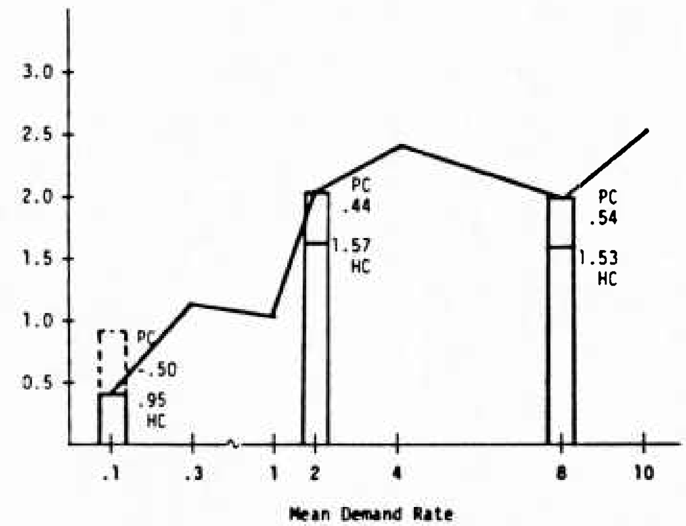


IV - 22

APPENDIX IV

Categorization of Total Cost Savings

Total Cost Savings In Dollars per Period



$1 - \Phi(\tau-1) = .05$

$\sigma^2:\mu = 1:1$

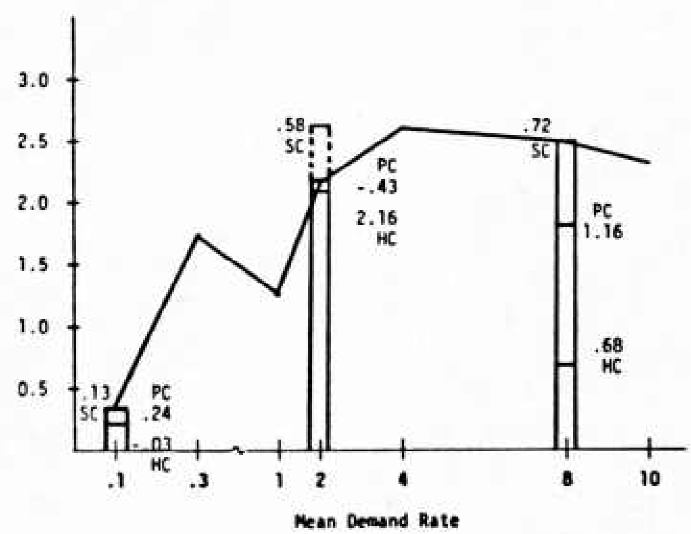
R = .99

L = 4

K = 0

IV - 23

Total Cost Savings In Dollars per Period



$1 - \Phi(\tau-1) = .05$

$\sigma^2:\mu = 1:1$

R = .99

L = 4

K = 64

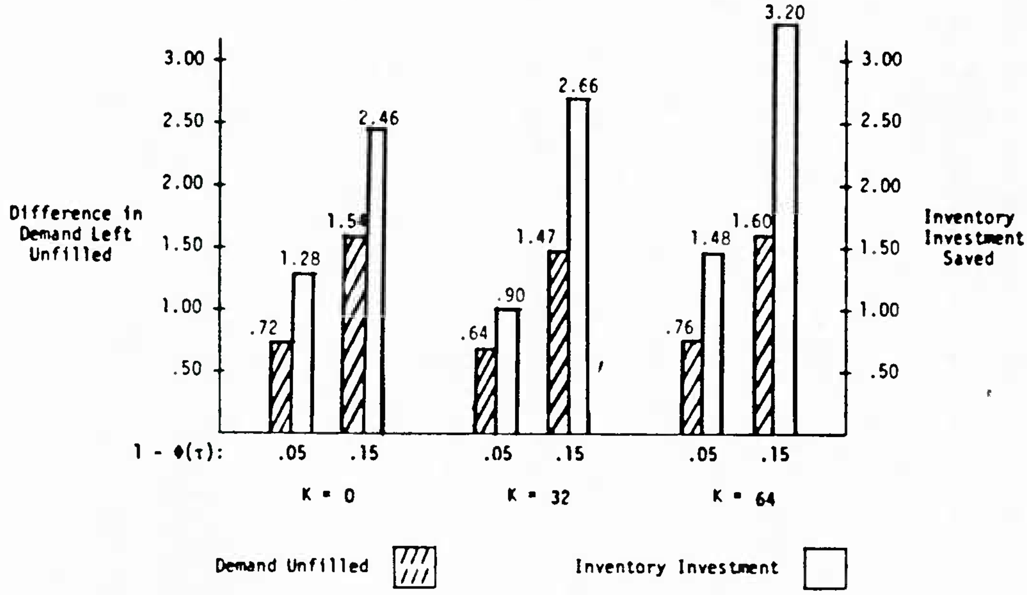
IV - 24

APPENDIX V

UNFILLED DEMAND VS THE REDUCTION IN INVENTORY INVESTMENT

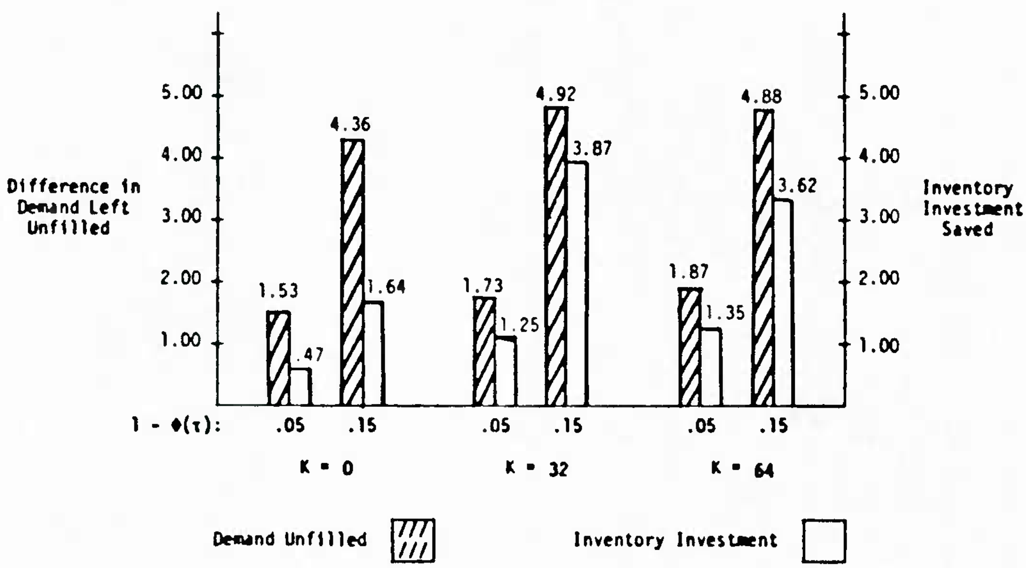
Unfilled Demand vs the Reduction in Inventory Investment

$\mu = 4$ $\sigma^2:\mu = 9:1$ $R = .8$ $L = 0$



V - 1

$\mu = 16$ $\sigma^2:\mu = 9:1$ $R = .8$ $L = 0$

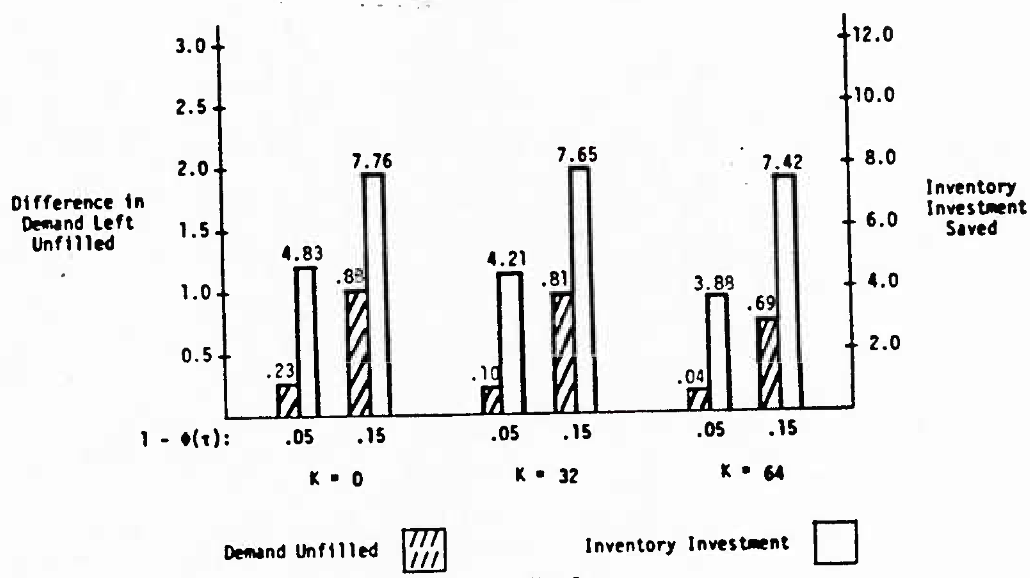


V - 2

APPENDIX V

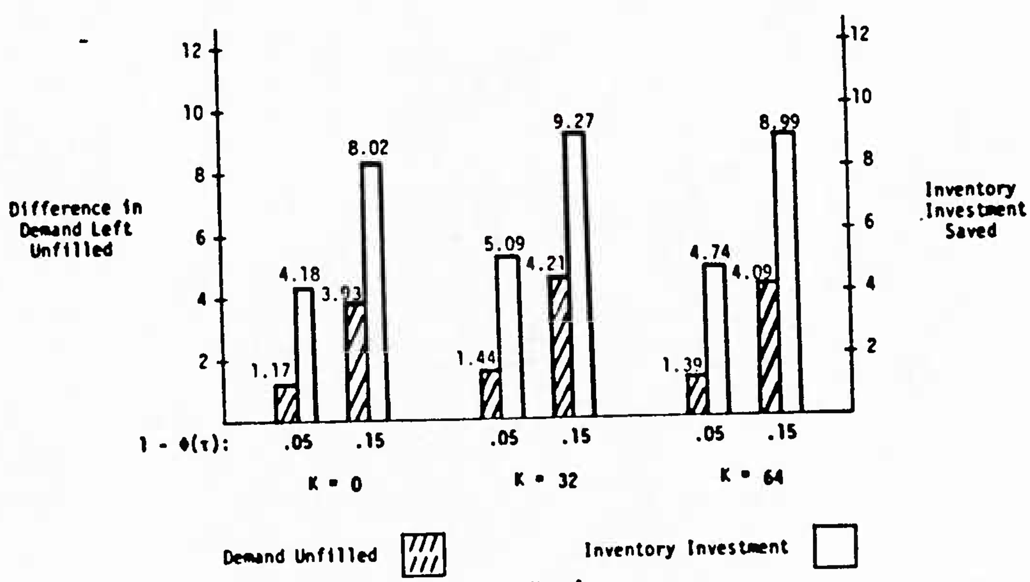
Unfilled Demand vs the Reduction in Inventory Investment

μ = 4 $\sigma^2:\mu$ = 9:1 R = .8 L = 4



V - 3

μ = 16 $\sigma^2:\mu$ = 9:1 R = .8 L = 4

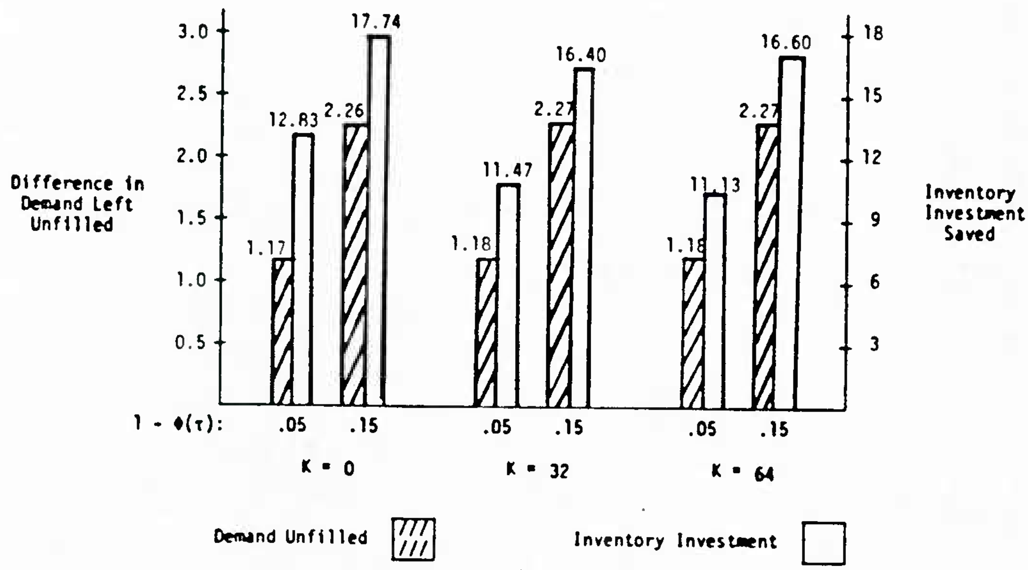


V - 4

APPENDIX V

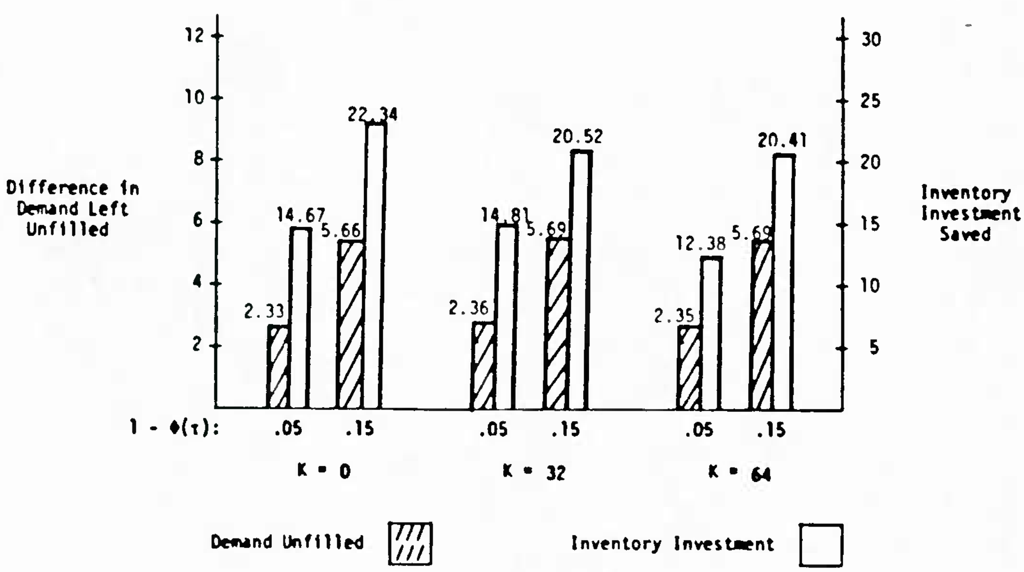
Unfilled Demand vs the Reduction in Inventory Investment

$\mu = 4$ $\sigma^2:\mu = 9:1$ $R = .99$ $L = 0$



V - 5

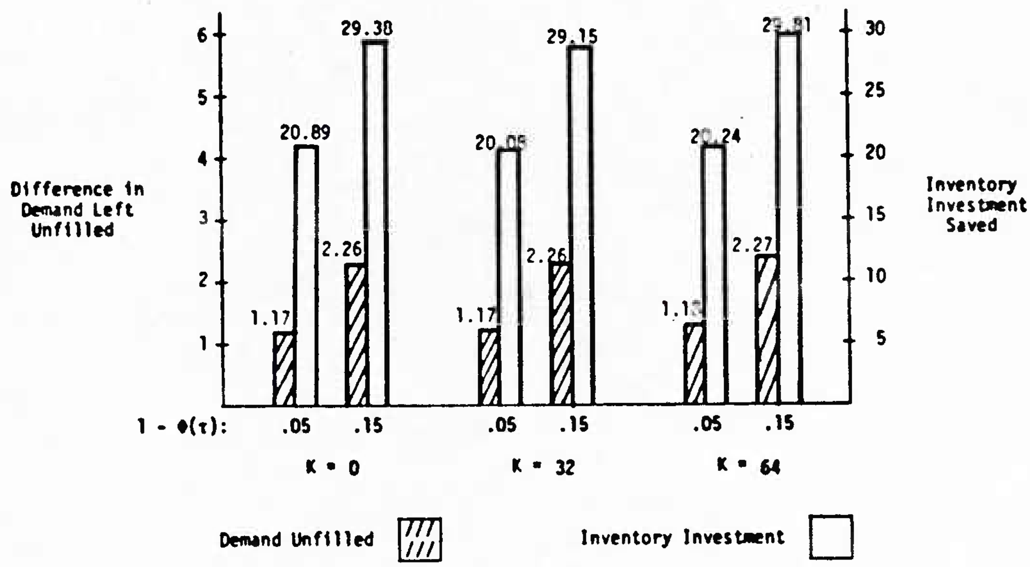
$\mu = 16$ $\sigma^2:\mu = 9:1$ $R = .99$ $L = 0$



APPENDIX V

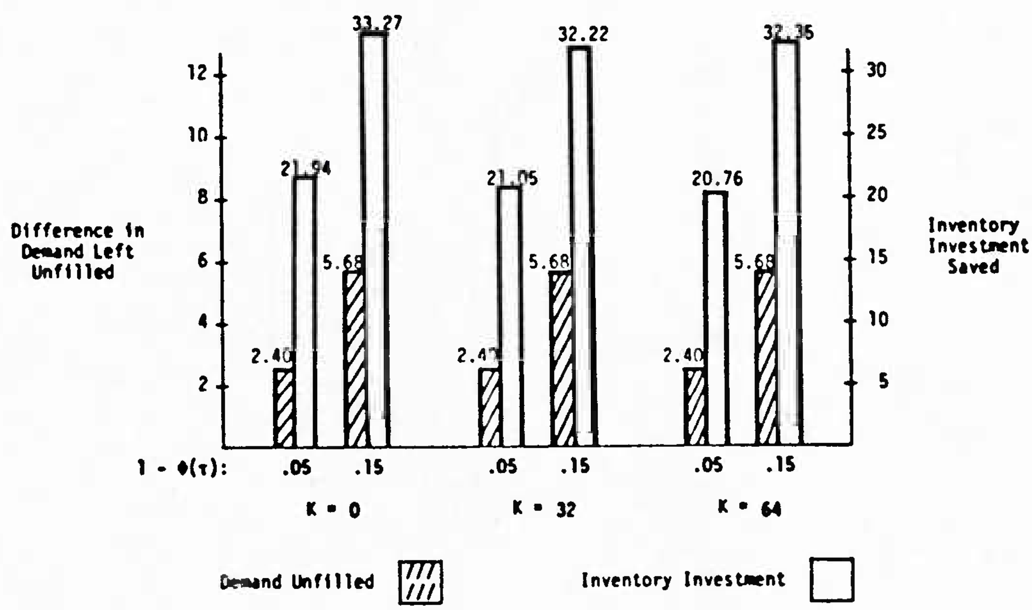
Unfilled Demand vs the Reduction in Inventory Investment

$\mu = 4$ $\sigma^2:\mu = 9:1$ $R = .99$ $L = 4$



V - 7

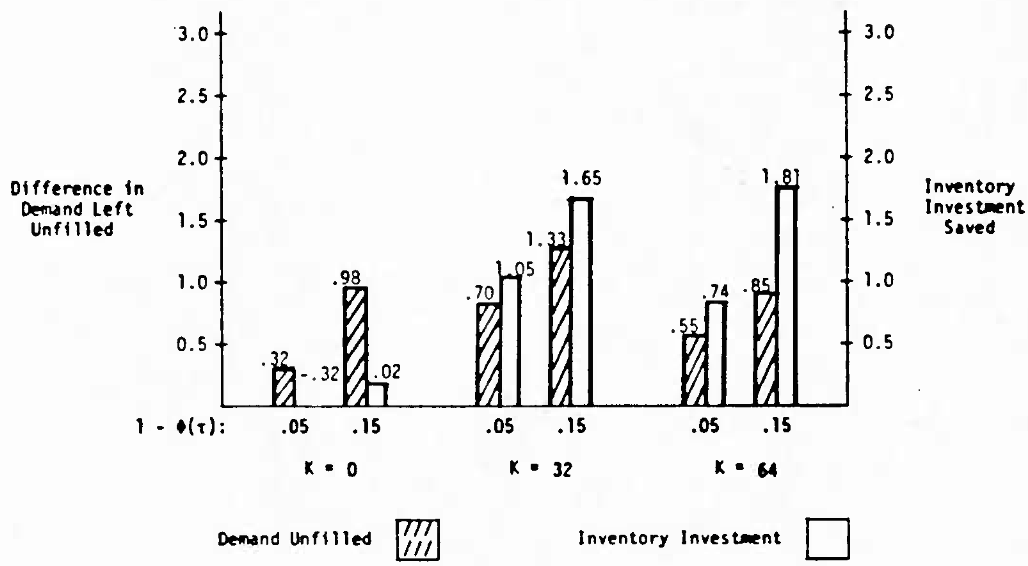
$\mu = 16$ $\sigma^2:\mu = 9:1$ $R = .99$ $L = 4$



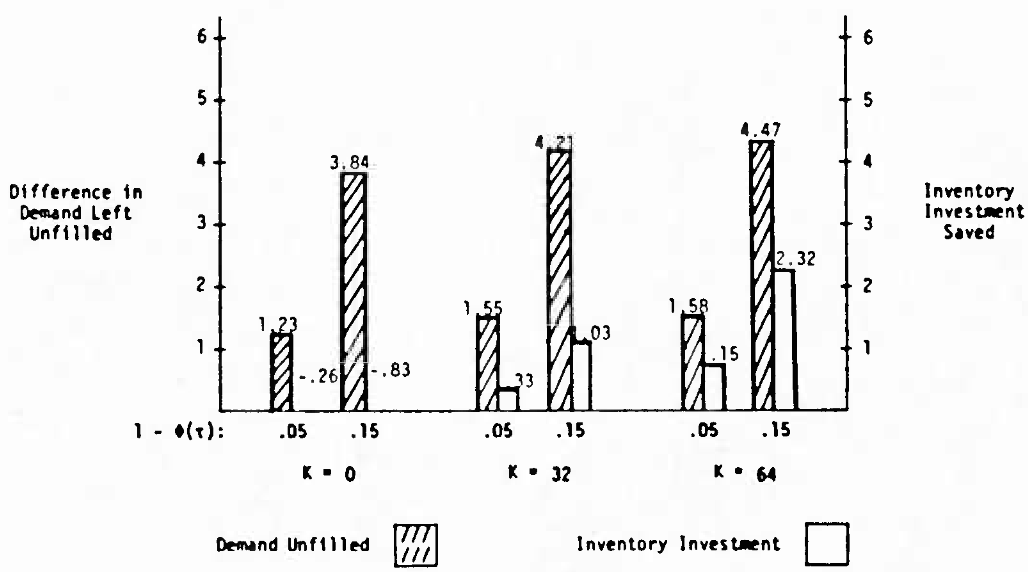
V - 8

Unfilled Demand vs the Reduction in Inventory Investment

$\mu = 4$ $\sigma^2:\mu = 3:1$ $R = .8$ $L = 0$



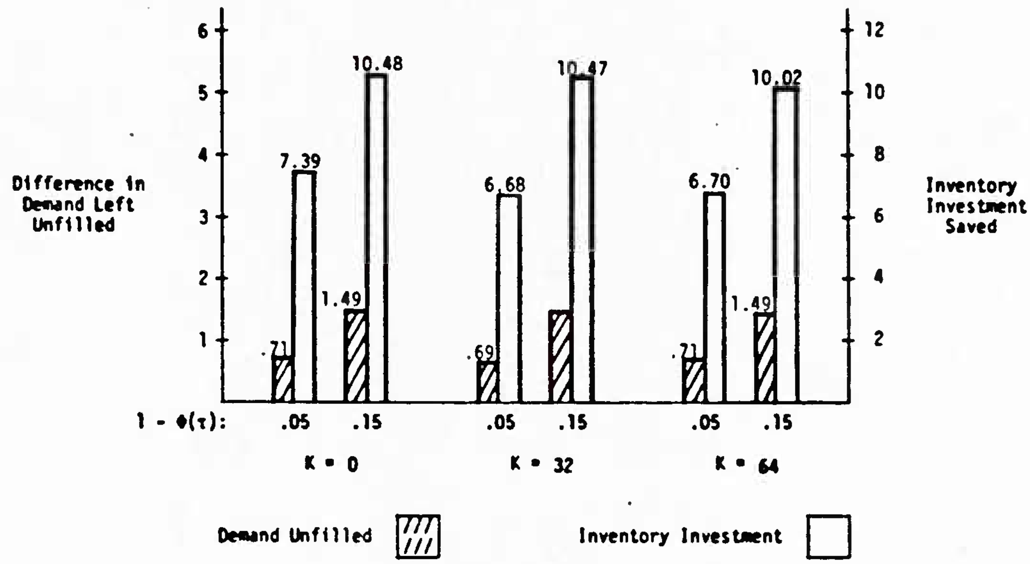
$\mu = 16$ $\sigma^2:\mu = 3:1$ $R = .8$ $L = 0$



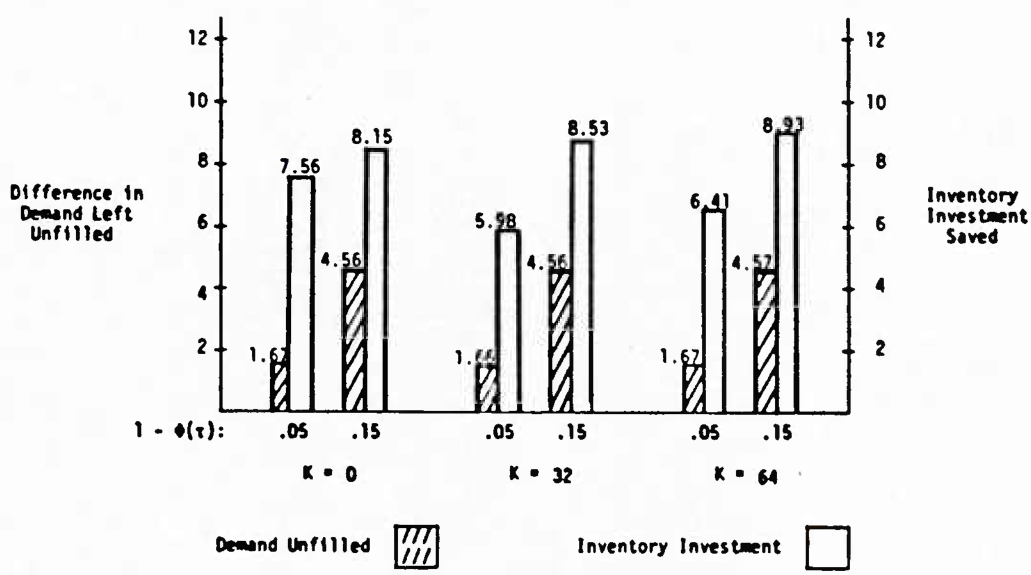
APPENDIX V

Unfilled Demand vs the Reduction in Inventory Investment

$\mu = 4$ $\sigma^2 : \mu = 3:1$ $R = .99$ $L = 4$

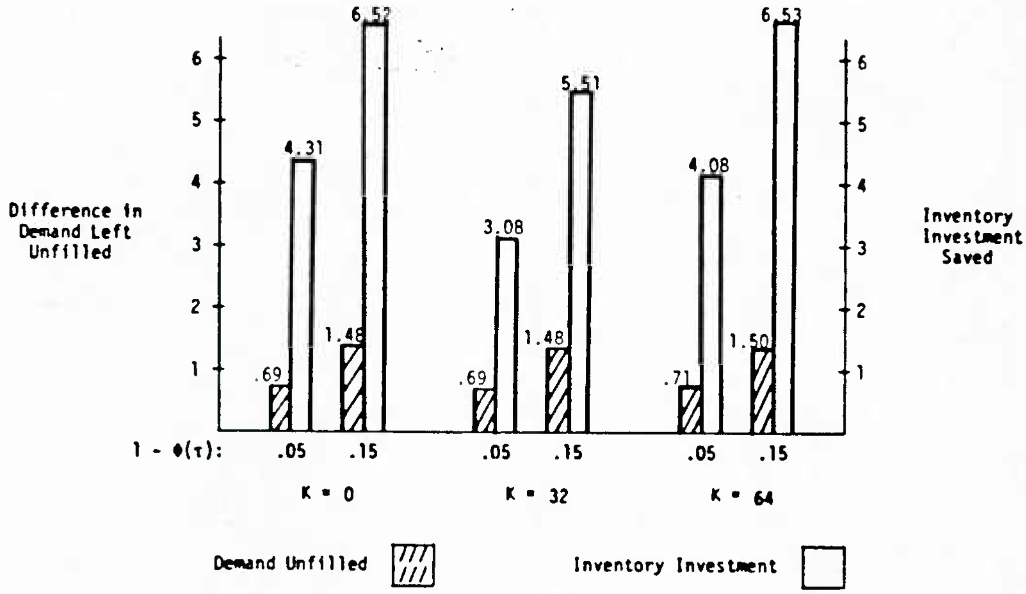


$\mu = 16$ $\sigma^2 : \mu = 3:1$ $R = .99$ $L = 4$

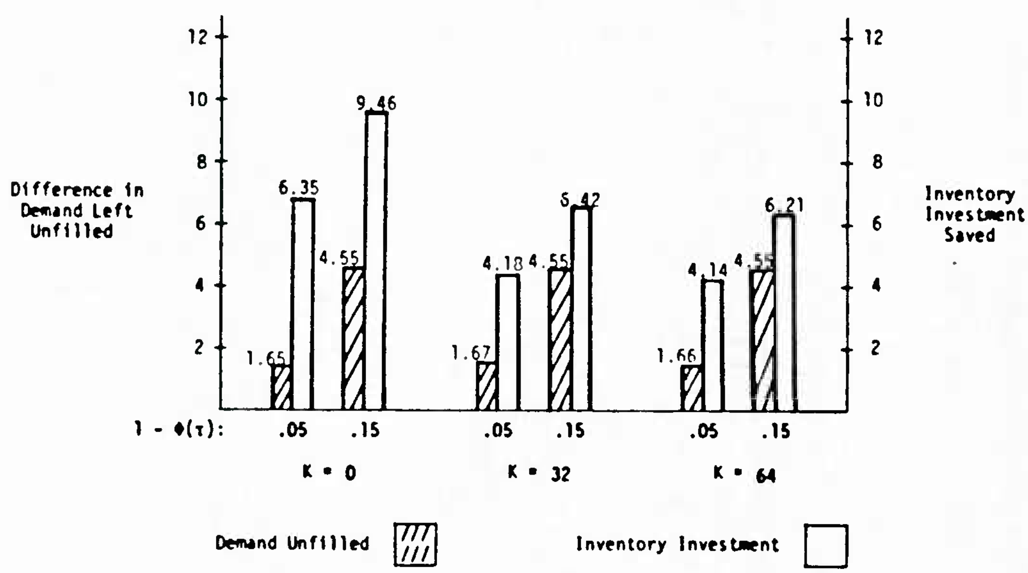


Unfilled Demand vs the Reduction in Inventory Investment

μ = 4 $\sigma^2:\mu$ = 3:1 R = .99 L = 0



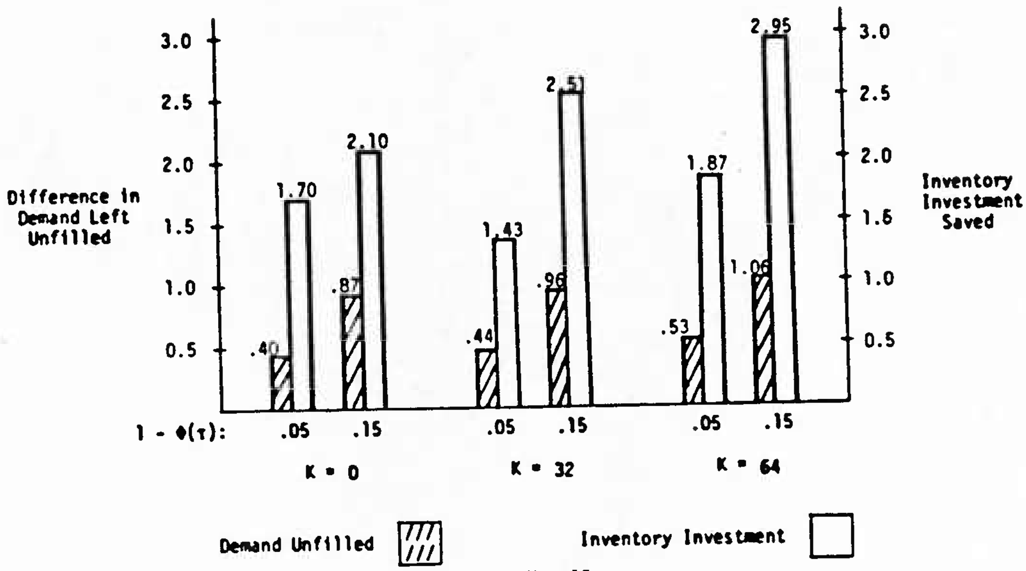
μ = 16 $\sigma^2:\mu$ = 3:1 R = .99 L = 0



APPENDIX V

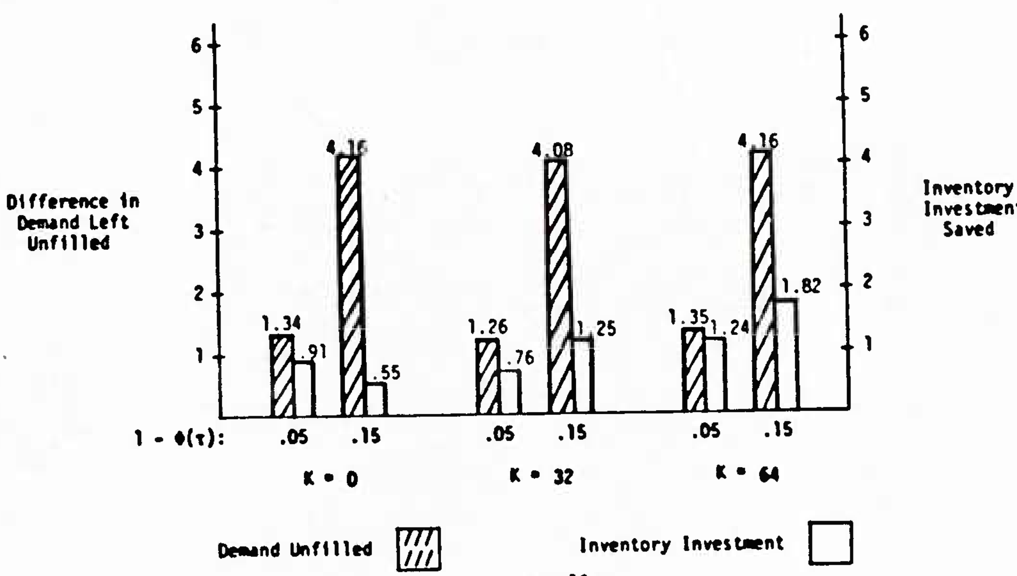
Unfilled Demand vs the Reduction in Inventory Investment

$\mu = 4$ $\sigma^2:\mu = 3:1$ $R = .8$ $L = 4$



V - 15

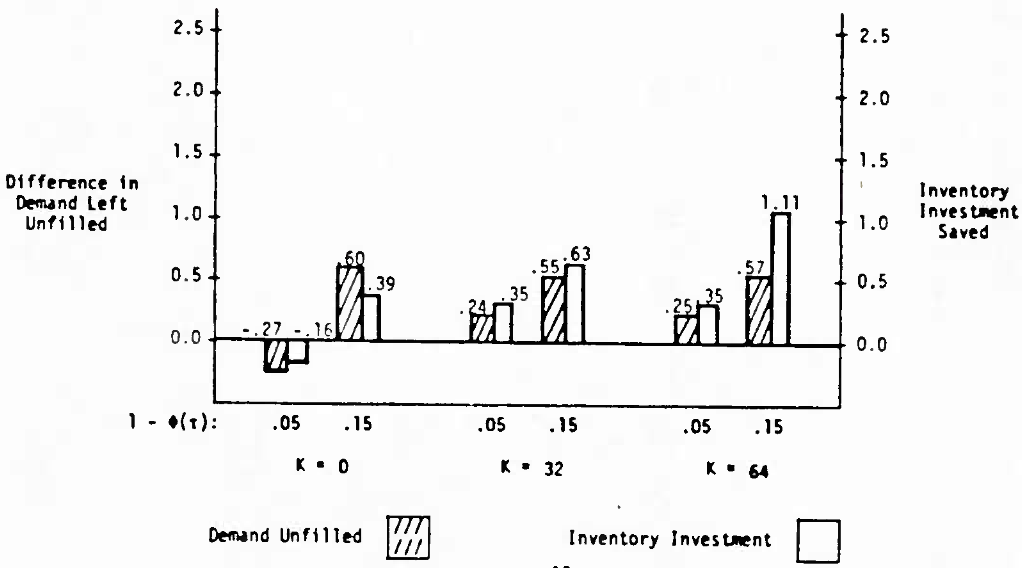
$\mu = 16$ $\sigma^2:\mu = 3:1$ $R = .8$ $L = 4$



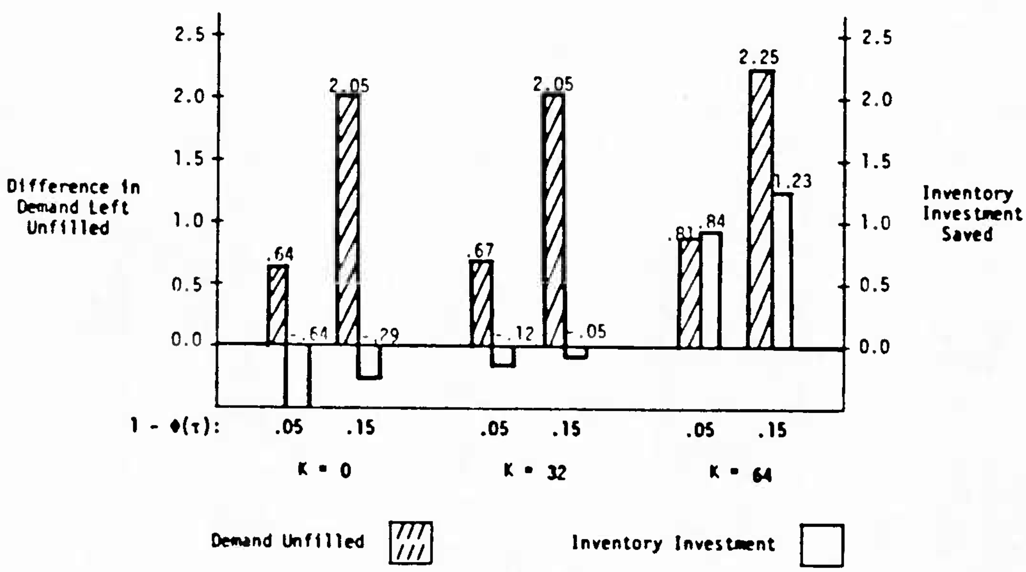
V - 16

Unfilled Demand vs the Reduction in Inventory Investment

μ = 2 $\sigma^2:\mu$ = 1:1 R = .8 L = 0



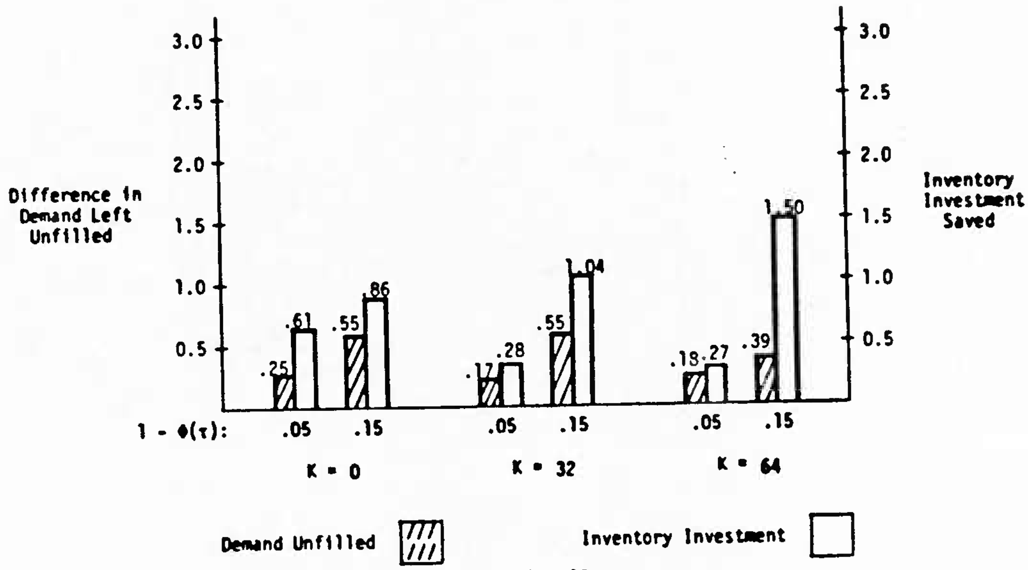
μ = 8 $\sigma^2:\mu$ = 1:1 R = .8 L = 0



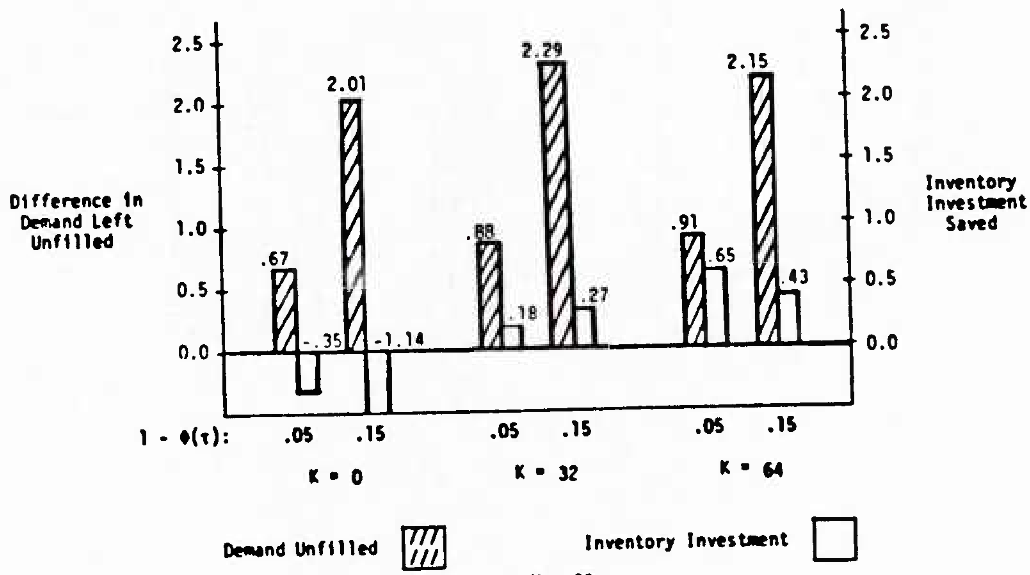
APPENDIX V

Unfilled Demand vs the Reduction in Inventory Investment

$\mu = 2$ $\sigma^2:\mu = 1:1$ $R = .8$ $L = 4$

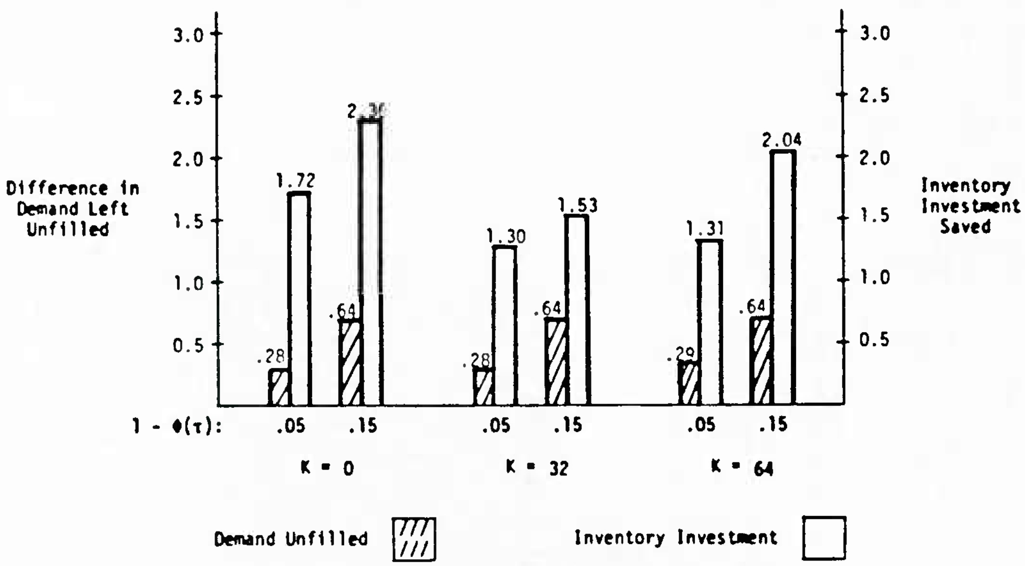


$\mu = 8$ $\sigma^2:\mu = 1:1$ $R = .8$ $L = 4$

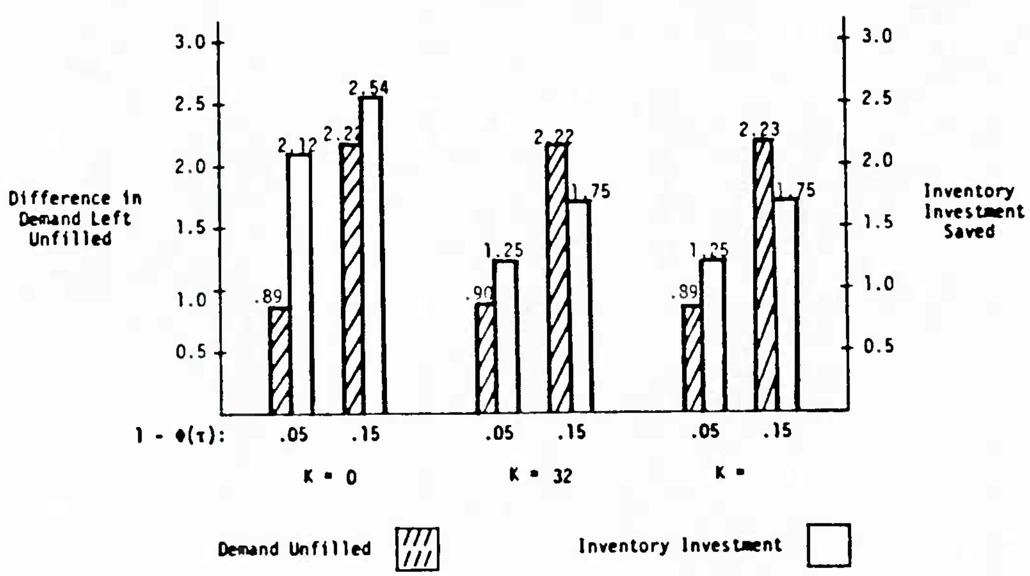


Unfilled Demand vs the Reduction in Inventory Investment

μ = 2 $\sigma^2:\mu$ = 1:1 R = .99 L = 0



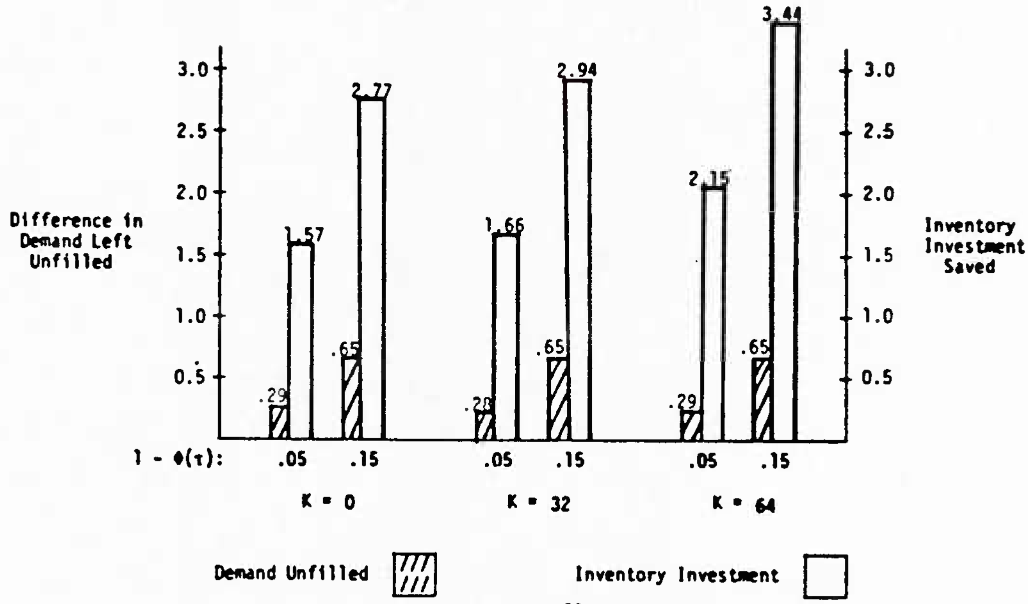
μ = 8 $\sigma^2:\mu$ = 1:1 R = .99 L = 0



APPENDIX V

Unfilled Demand vs the Reduction in Inventory Investment

$\mu = 2$ $\sigma^2:\mu = 1:1$ R = .99 L = 4



$\mu = 8$ $\sigma^2:\mu = 1:1$ R = .99 L = 4